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Inflation Rate and Nominal Exchange Rate Volatility brought about by Optimal Monetary Policy under Local Currency Pricing*

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Abstract

We analyze the fluctuations in inflation and nominal exchange rates under optimal monetary policy with local currency pricing by developing a two-country model using dynamic stochastic general equilibrium (DSGE) with local currency pricing (LCP) and comparing fluctuations under LCP with fluctuations under producer currency pricing (PCP). Although previous DSGE literature that assumes PCP finds stabilizing domestic inflation is optimal when the aim is to minimize welfare costs, we show that completely stabilizing consumer price index (CPI) inflation is optimal from that viewpoint. In addition, we show that completely stabilizing CPI inflation is equivalent to completely stabilizing the nominal exchange rate.

Keywords: Local Currency Pricing, DSGE, Optimal Monetary Policy, CPI Inflation, Fixed Exchange Rate

JEL Classification: E52; E62; F41

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1 Introduction

What kind of inflation rate should be stabilized from the viewpoint of minimizing welfare costs? Which exchange rate regime should be chosen from that viewpoint? These are important questions when discussing optimal monetary policy in an open economy. This paper shows that consumer price index (CPI) inflation should be stabilized if we assume that prices are set in consumers' currency (denoted as local currency pricing, LCP). In addition, we show that stabilizing the CPI inflation rate is not inconsistent with fixed exchange rates under LCP. Our finding contrasts with preceding papers discussing optimal monetary policy utilizing the dynamic stochastic general equilibrium (DSGE) framework, because those papers show that the producer price index (PPI) inflation rate should be stabilized from that viewpoint even if an open economy is assumed.[1] Gali and Monacelli (2005) show that optimal monetary policy in a small open economy is consistent with PPI inflation targeting.[2] Although they do not mention it explicitly, they assume that prices are set in producers' currency (denoted producer currency pricing, PCP). They compare three policy regimes (PPI-inflation-based and CPI-inflation-based Taylor rules and fixed exchange rate regimes) and show that a PPI-inflation-based Taylor rule produces the most similar macroeconomic volatility, which is brought about by optimal monetary policy. In addition, their policy implication implies that outcomes of optimal monetary policy are not fundamentally different from those of a closed economy. While they do not highlight the firms' price-setting behavior, Gali and Monacelli (2005) imply that PPI inflation targeting is optimal under PCP. Developing a two-country model, Benigno and Benigno (2006) implicitly show that stabilizing PPI inflation minimizes welfare costs under PCP.

[1] In many DSGE papers, there are no distinctions between the definitions of PPI inflation, domestic inflation, and GDP inflation.

[2] Correctly, PPI inflation is designated domestic inflation in Gali and Monacelli (2005). However, their definition of domestic inflation is consistent with our definition of PPI inflation.

To find the sort of inflation rate that should be stabilized under LCP, we develop an LCP model assuming two countries at first. In addition, to clarify what determines differences in policy implications of LCP, we also develop a two-country model with PCP, which is assumed by canonical preceding papers such as Gali and Monacelli (2005) and Benigno and Benigno (2006). Goods markets are completely segmented and there is no international arbitrage trade in our LCP model while Monacelli (2005) introduces importers following LCP to generate imperfect pass-through environment. We derive well microfounded loss functions under both LCP and PCP, stemming from the second-order Taylor expanded utility function following Woodford (2003) and Gali (2008). We assume that central banks in two countries solve the optimization problem under both LCP and PCP, and impulse response functions (IRFs) are calculated. We calculate IRFs under the special case in which the relative risk aversion and the elasticity of substitution between goods produced in both countries are unity and under the general case in which the relative risk aversion and the elasticity of substitution between goods produced in both countries are 3 and 4.5, respectively.[3] Note that the elasticity settings in the special case are consistent with Gali and Monacelli's (2005) settings, and the elasticity settings in the general case are consistent with Benigno and Benigno's (2006). To enable comparison with the results of Gali and Monacelli (2005) and to discuss optimal monetary policy under general parameterization, we analyze two cases. Because we are interested in macroeconomic volatility that impacts on welfare costs based on a second-order approximated utility function, and are also interested in nominal exchange rate volatility under PCP and LCP, we calculate macroeconomic volatility including the nominal exchange rate, varying the relative risk aversion and the elasticity of substitution between goods produced in both countries.

[3]The relative risk aversion and the elasticity of substitution between goods produced in both countries are often dubbed the intertemporal elasticity of substitution and the intratemporal elasticity of substitution, respectively.

Finally, we calculate welfare costs, varying the relative risk aversion and the elasticity of substitution between goods produced in both countries.

We summarize our results as follows. We show that optimal monetary policy under LCP stabilizes the CPI inflation rate completely, but does not stabilize the PPI inflation rate. Roughly speaking, optimal monetary policy under LCP is CPI inflation targeting. This result is quite different from the result in Gali and Monacelli (2005). Our result is confirmed by IRFs, volatility of CPI inflation, and the loss function stemming from the second-order approximated utility function. Interestingly, the quadratic terms of the CPI inflation rate appear in our loss function and replace the quadratic terms of PPI inflation under LCP, although the quadratic terms of PPI inflation appear in our loss function under PCP as in Gali and Monacelli (2005) and Benigno and Benigno (2006). In addition, we show that optimal monetary policy under LCP is not consistent with a fixed exchange rate regime. Not only volatility of CPI inflation, but also volatility of nominal exchange rate are completely stabilized in our LCP model. This result is quite different from Gali and Monacelli (2005). That is, optimal monetary policy under LCP is not only consistent with CPI inflation targeting but also consistent with a fixed exchange rate.

Now we compare our results with some literature introducing LCP into the model. Motivated by external shocks hitting an emerging market via changes in interest rates and the terms of trade (TOT), Devereux, Lane and Xu (2006) show that CPI price stability maximizes households' utility by introducing financial frictions, introducing a nontraded sector, and by importers adopting LCP, and by comparing with nontradable goods price stability and a fixed exchange rate regime. Focusing on policy trade-offs between stabilization of domestically produced goods prices and stabilization of imported goods prices, Corsetti, Dedola and Leduc (2007) mention that CPI stabilization is optimal from the viewpoint of maximizing households' utility. Their LCP model is quite sophisticated because of introducing both upstream and downstream firms to generate policy trade-offs between stabilization of domestically produced goods price and stabilization of imported goods price. They compare various policy regimes and show that strict CPI inflation targeting is close to optimal monetary policy for reducing volatility of the CPI inflation rate and maximizing real GDP. Devereux and Engel (2003) mention the exchange rate regime and show that a fixed exchange rate regime minimizes welfare costs with their two-country model with LCP firms. However, those papers do not reconcile with each other and there is an inconsistency between the results of those papers. Devereux, Lane and Xu (2006) and Dedola and Leduc (2007) show that there is a trade-off between CPI stabilization and nominal exchange rate stabilization and imply that CPI stabilization results in stabilization of nominal exchange rates, which is inconsistent with Devereux, Lane and Xu (2006). Our result is consistent with none of the above papers, because we show that optimal monetary policy under LCP stabilizes both CPI inflation rate and nominal exchange rate and reconcile the inconsistency among those papers. Our result contrasts with Monacelli's (2005) result. Although he does not focus on the sort of inflation rate which should be targeted, he develops a low pass-through model stemming from LCP and shows the difficulty of dissolving inflation–output gap trade-offs simultaneously because the low of one price (LOOP) gap, namely deviations from LOOP, appears in his New Keynesian Philips Curve (NKPC) and LOOP gap as if it plays the role of cost-push shocks. Our result implies that zero welfare cost is attained through completely dissolving inflation–output gap trade-offs by optimal monetary policy, which is consistent with CPI inflation targeting. By appropriately choosing the rate of targeted inflation, we can avoid welfare costs.

The rest of this paper is organized as follows. Section 2 derives two models, the LCP and the PCP model. Section 3 analyzes optimal monetary policy by deriving welfare costs and FONCs for a central bank with commitment and calibration. Section 4 analyzes effects on macroeconomic volatility and welfare costs of varying relative risk aversion and the elasticity of substitution between goods produced in two countries. Section 5 concludes the paper. An appendix discusses international monetary policy cooperation between two countries, omitted in the text because there we focus on fluctuations in inflation and nominal exchange rates brought about by optimal monetary policy.

2 The Model

We construct a two-country model belonging to the class of DSGE models with nominal rigidities and imperfect competition, basically following Gali and Monacelli (2005). We alter Gali and Monacelli's (2005) small open-economy model to a two-country economy model following Obstfeld and Rogoff (1995), although we assume all goods are tradable. The union-wide economy consists of two countries, H and F. Country H produces an array of differentiated goods indexed by the interval $h \in [0,1]$, while country F produces an array of differentiated goods indexed by $f \in [1,2]$. In addition, we derive the two models, LCP and PCP.

Note that we take a definition $v_t \equiv \ln\left(\frac{V_t}{V}\right)$ if there are no provisions where V_t denotes an arbitrary variable and V denotes the steady state value of V_t.

2.1 LCP Model

Under LCP, goods markets are completely segmented and there is no international arbitrage trade in our LCP model. LOOP is not necessarily applied, because firms can choose prices to sell goods in countries H and F separately. Thus, $P_t(h) = \mathcal{E}_t P_t^*(h)$ and $P_t(f) = \mathcal{E}_t P_t^*(f)$, hence $P_{H,t} = \mathcal{E}_t P_{H,t}^*$ and $P_{F,t} = \mathcal{E}_t P_{F,t}^*$ do not necessarily hold where $P_t(h)$ and $P_t(f)$ denote the price of a generic good produced in country H in terms of country H's currency, $P_{H,t} \equiv \left[\int_0^1 P_t(h)^{1-\varepsilon} dh\right]^{\frac{1}{1-\varepsilon}}$ and $P_{F,t} \equiv \left[\int_1^2 P_t(f)^{1-\varepsilon} dh\right]^{\frac{1}{1-\varepsilon}}$ denote indices of the price of generic goods produced in countries H and F, respectively, $\mathcal{E}_t$ denotes the nominal exchange rate.[4] Note that quantities and prices particular to country F are denoted by asterisks, while quantities and prices without asterisks are those in country H.

2.1.1 Households

The preferences of the representative household in country H are given by:

$$\mathcal{U} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U_t, \tag{1}$$

where $U_t \equiv \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} N_t^{1+\varphi}$ denotes the period utility, E_t denotes the expectation, conditional on the information set at period t, $\beta \in (0,1)$ denotes the subjective discount factor, C_t denotes consumption, $N_t \equiv \int_0^1 N_t(h)\, dh$ denotes hours of work, σ denotes the relative risk aversion and φ denotes the inverse of the labor supply elasticity. The preferences of the representative household in country F are defined analogously.

More precisely, private consumption is a composite index defined by:

$$C_t \equiv \left[\left(\frac{1}{2}\right)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \left(\frac{1}{2}\right)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $C_{H,t} \equiv \left[\int_0^1 C_t(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}$ and $C_{F,t} \equiv \left[\int_1^2 C_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}$ denote Dixit–Stiglitz-type indices of consumption across the home goods and foreign goods, respectively, and $\eta > 0$ denotes the elasticity of substitution between goods produced in countries H and F. Note that C_t^* is defined analogously to Eq.(2).

Total consumption expenditures by households in country H are given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_t C_t$. A sequence of budget constraints in country H is given by:

$$B_t + W_t N_t - T_t \geq P_t C_t + \mathrm{E}_t \left(Q_{t,t+1} B_{t+1} \right), \tag{3}$$

[4] By citing Betts and Devereux (2000), Mark (2001) clearly explains LCP.

where $Q_{t,t+1}$ denotes the stochastic discount factor, B_t denotes the nominal payoff of the bond portfolio purchased by households, W_t denotes the nominal wage, and T_t denotes lump-sum taxes. The budget constraint in country F is given analogously. Furthermore:

$$P_t \equiv \left(\frac{1}{2} P_{H,t}^{1-\eta} + \frac{1}{2} P_{F,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}, \tag{4}$$

denotes the CPI. P_t^* is defined analogously to this equality. Log-linearizing this equality yields $p_t = \frac{1}{2} p_{H,t} + \frac{1}{2} p_{F,t}$, which implies as follows:

$$\pi_t = \frac{1}{2} \pi_{H,t} + \frac{1}{2} \pi_{F,t}, \tag{5}$$

where $\pi_t \equiv p_t - p_{t-1}$ denotes CPI inflation with $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ and $\pi_{F,t} = p_{F,t} - p_{F,t-1}$.

The optimal allocation of any given expenditure within each category of goods implies the demand functions as follows:

$$\begin{aligned} C_t(h) &= \left(\frac{P_t(h)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \quad ; \quad C_t(f) = \left(\frac{P_t(f)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t} \\ C_t^*(h) &= \left(\frac{P_t^*(h)}{P_{H,t}^*} \right)^{-\varepsilon} C_{H,t}^* \quad ; \quad C_t^*(f) = \left(\frac{P_t^*(f)}{P_{F,t}^*} \right)^{-\varepsilon} C_{F,t}^*. \end{aligned} \tag{6}$$

The optimal allocation of expenditures between domestic and foreign goods is given by:

$$\begin{aligned} C_{H,t} &= \frac{1}{2} \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad ; \quad C_{F,t} = \frac{1}{2} \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t, \\ C_{H,t}^* &= \frac{1}{2} \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^* \quad ; \quad C_{F,t}^* = \frac{1}{2} \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\eta} C_t^*. \end{aligned} \tag{7}$$

The representative household maximizes Eq.(1) subject to Eq.(3). The optimality conditions are given by:

$$R_t \beta \mathrm{E}_t \left(\frac{C_{t+1}^{-\sigma} P_t}{C_t^{-\sigma} P_{t+1}} \right) = 1, \tag{8}$$

which is a conventional Euler equation and

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t}, \tag{9}$$

which is a standard intratemporal optimality condition where $R_t \equiv 1 + r_t$ satisfying $R_t^{-1} = \mathrm{E}_t Q_{t,t+1}$ denotes the gross nominal return on a riskless one-period discount bond paying off one unit of the common currency (in short, the gross nominal interest rate), and r_t denotes the net nominal interest rate. Eq.(8) is an intertemporal optimality condition, namely the Euler equation, and Eq.(9) is an intratemporal optimality condition. Optimality conditions in country F are given analogously.

Log-linearizing Eq.(8), we obtain:

$$c_t = \mathrm{E}_t c_{t+1} - \frac{1}{\sigma}\hat{r}_t + \frac{1}{\sigma}\mathrm{E}_t \pi_{t+1} \tag{10}$$

with $\hat{r}_t \equiv \ln\left(\frac{R_t}{R}\right)$.

The uncovered interest rate parity (UIP) is applied between countries H and F and is given by:

$$R_t = R_t^* \mathrm{E}_t \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)$$

with $R_t^* \equiv 1 + r_t^*$. Log-linearizing the UIP, we have this familiar expression:

$$\mathrm{E}_t \left(\Delta e_{t+1}\right) = \hat{r}_t - \hat{r}_t^*,$$

with $\Delta v_t \equiv v_t - v_{t-1}$ and $e_t \equiv \ln\left(\frac{\mathcal{E}_t}{\mathcal{E}}\right)$.

Combining Eq.(8) and the UIP and iterating with different initial conditions, we have the following optimal risk-sharing condition:

$$C_t^\sigma = \vartheta \left(C_t^*\right)^\sigma \mathcal{Q}_t,$$

with $\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$ denoting the real exchange rate and ϑ denoting a constant depending on the initial value. Log-linearizing this equality, we have:

$$c_t = c_t^* + \frac{1}{\sigma} q_t. \tag{11}$$

Our setting is definitely different from that in Betts and Devereux (2000), who introduce pricing-to-market behavior, which is consistent with LCP in our definition, into the Redux model developed by Obstfeld and Rogoff (1995). Although we and Betts and Devereux (2000) both allow violations of the LOOP, purchasing power parity (PPP) is applied in our paper but not necessarily in their paper. While we assume international risk-sharing conditions as shown in Eq.(11), Betts and Devereux (2000) introduce restricting asset availability, which inhibits international optimal risk sharing.[5] Because of their setting, Eq.(11) is no longer applied, hence PPP is not necessarily applied. In addition, this implies that violation of PPP does not stem from LCP. We further discuss the relationship between LOOP and PPP in Section 2.1.3.

2.1.2 Market Clearing

The market for tradables and nontradables in country H clears when domestic demand equals domestic supply, as follows:

$$Y_t(h) = C_t(h) + C_t^*(h), \tag{12}$$

where $Y_t(h)$ denotes the output of good h, which is the market clearing condition. The market clearing condition in country F is analogous. Plugging Eq.(6) into Eq.(12) yields:

$$Y_t(h) = \frac{1}{2}\left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \frac{1}{2}\left(\frac{P_t^*(h)}{P_{H,t}^*}\right)^{-\varepsilon}\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^*. \tag{13}$$

Let $Y_t \equiv \left[\int_0^1 Y_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}$ represent the index for aggregate output in country H. Under LCP, we obtain:

$$\begin{aligned} Y_t &= \frac{1}{2}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \frac{1}{2}\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^*, \\ &= \frac{1}{2}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[1 + \left(\frac{P_{H,t}}{P_t}\right)^{\eta}\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} \mathcal{Q}_t^{-\frac{1}{\sigma}}\right], \end{aligned} \tag{14}$$

by combining Eqs.(13), Dixit–Stiglitz aggregators for output and prices, where we take the optimal risk-sharing condition in the second line in Eq.(14).

[5] Although they develop a two-country model that consists of home and foreign countries, an internationally tradable asset is denominated in the home country's currency.

We define the terms of trade (TOT) as follows:

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{\mathcal{E}_t P^*_{H,t}}, \tag{15}$$

where $\mathcal{S}_t$ is foreign TOT. The numerator is the export price of goods produced in country F in terms of country H's currency and the denominator is the export price of goods produced in country F in terms of country H's currency. Log-linearizing Eq.(15), we have:

$$s_t = p_{F,t} - e_t - p^*_{H,t}, \tag{16}$$

with $s_t \equiv \ln\mathcal{S}_t$.

Plugging Eq.(16) into log-linearized Eq.(14), we have:

$$y_t = c_t + \frac{\eta}{2}s_t + \frac{1}{2}\left(\eta - \frac{1}{\sigma}\right)q_t,$$

which is log-linearized market clearing in country H under LCP. There is a difference between this equality and Eq.(42), which is the counterpart of this equality under PCP because the logarithmic real exchange rate q_t appears in this equality. However, this equality boils down to Eq.(42) because PPP is applied, which implies that $q_t = 0$, although we assume LCP. We discuss PPP under LCP in Section 2.3.

Combining Eq.(42) and its counterpart in country F, we have:

$$s_t = \frac{1}{\eta}\left(y_t - y^*_t\right) - q_t,$$

which clarifies the relationship between the TOT and relative output under LCP. As mentioned, $q_t = 0$ is applied, although we assume LCP. Hence, this equality boils down to Eq.(41), which is the counterpart of this equality under PCP.

2.1.3 Firms

Each producer uses a linear technology to produce a differentiated good as follows:

$$Y_t(h) = A_t N_t(h), \tag{17}$$

where A_t denotes stochastic productivity in country H. Firms in country F have a technology analogous to firms in country H.

Using Dixit–Stiglitz aggregators, Eq.(17) can be rewritten as:

$$N_t = \frac{Y_t D_t}{A_t}, \tag{18}$$

with $D_t \equiv \int_0^1 \frac{Y_t(h)}{Y_t} dh$. Because d_t is $o\left(\| \xi \|^2\right)$, a first-order approximation of this equality is given by:

$$y_t = a_t + n_t, \tag{19}$$

which is consistent with Gali and Monacelli's (2005) log-linearized production function.

As in many DSGE papers, including Gali and Monacelli (2005), we assume that firms set prices using Calvo–Yun-style price-setting behavior. Hence, a fraction $1 - \theta$ of firms set new prices each period, with an individual firm's probability of re-optimizing in any given period being independent of the time elapsed since it last set its prices. Each producer produces a single differentiated good and prices it to reflect the elasticity of substitution across goods produced given the CPI. This is because each firm plays an active part in the monopolistically competitive market. In addition, we assume that firms have the ability to engage in price discrimination by setting a domestic price in terms of domestic currency for domestic sales that differs from the price that it sets for exports. This is LCP behavior. Under Calvo–Yun-style price-setting behavior and LCP behavior in a monopolistically competitive market, the maximization problems that producers in country H face are as follows:

$$\max_{\tilde{P}_{H,t},\tilde{P}^*_{H,t}} \sum_{k=0}^{\infty}\theta^k \mathrm{E}_t \left\{ Q_{t,t+k}\left[\tilde{P}_{H,t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} C_{H,t+k} + \mathcal{E}_{t+k}\tilde{P}^*_{H,t}\left(\frac{\tilde{P}^*_{H,t}}{P^*_{H,t+k}}\right)^{-\varepsilon} C^*_{H,t+k} \right.\right.$$
$$\left.\left. - MC^n_{t+k}\left(\left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} C_{H,t+k} + \left(\frac{\tilde{P}^*_{H,t}}{P^*_{H,t+k}}\right)^{-\varepsilon} C^*_{H,t+k}\right)\right]\right\}, \quad (20)$$

where $\tilde{P}_{H,t}$ and $\tilde{P}^*_{H,t}$ are the prices chosen by firms when they obtain the chance to change prices associated with goods produced and sold in country H and goods produced in country H while sold in country F, respectively, $MC^n_t \equiv P_{P,t}MC_t$ denotes real marginal costs in country H, with $MC_t \equiv \frac{(1-\tau)W_t}{A_t P_{P,t}}$ and $P_{P,t}$ denotes the PPI in country H, which are defined as follows:

$$P_{P,t} \equiv \frac{P_{H,t}C_{H,t} + \mathcal{E}_t P^*_{H,t}C^*_{H,t}}{C_{H,t} + C^*_{H,t}},$$

which can be rewritten as $P_{P,t} = P_{H,t}$ when the LOOP is applied. The PPI in country F is defined analogously. By log-linearizing this equality, we have $p_{P,t} = \frac{1}{2}p_{H,t} + \frac{1}{2}\left(e_t + p^*_{H,t}\right)$, which implies the following:

$$\pi_{P,t} = \frac{1}{2}\pi_{H,t} + \frac{1}{2}\left(\Delta e_t + \pi^*_{H,t}\right), \quad (21)$$

where $\pi_{P,t}$ denotes the PPI inflation in country H and $\pi_{P,t} = \pi_{H,t}$ is applied when the LOOP is applied.

Note that the maximization problems that producers in country F face are analogous to Eq.(20). Because of nominal rigidities, Eq.(20) looks complicated. When there are no nominal rigidities, namely $\theta \to 0$, Eq.(20) problems boil down to:

$$\max_{P_{H,t},P^*_{H,t}} P_{H,t}C_{H,t} + \mathcal{E}_t P^*_{H,t}C^*_{H,t} - MC^n_t\left(C_{H,t} + C^*_{H,t}\right),$$

which implies that each firm sets its price in terms of the local currency in which each firm's good is sold and pay production costs in terms of the producer's currency.

Under LCP, we have multiple FONCs because firms can choose $\tilde{P}_{H,t}$ and $\tilde{P}^*_{H,t}$ separately. The FONCs for Eq.(20) are as follows:

$$\begin{aligned} \mathrm{E}_t\left[\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}\left(\tilde{P}_{H,t}-\zeta MC^n_{t+k}\right)\left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} C_{H,t+k}\right] &= 0, \\ \mathrm{E}_t\left[\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}\left(\tilde{P}^*_{H,t}\mathcal{E}_{t+k}-\zeta MC^n_{t+k}\right)\left(\frac{\tilde{P}^*_{H,t}}{P^*_{H,t+k}}\right)^{-\varepsilon} C^*_{H,t+k}\right] &= 0, \end{aligned}$$

which can be log-linearized as follows:

$$\begin{aligned} \tilde{p}_{H,t} &= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \mathrm{E}_t\left(mc^n_{t+k}\right), \\ \tilde{p}^*_{H,t} &= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \mathrm{E}_t\left(mc^n_{t+k}-e_{t+k}\right), \end{aligned} \tag{22}$$

with $\zeta \equiv \frac{\theta}{\theta-1}$ denoting a constant markup where we use the fact that $Q_{t,t+k} = \beta^k\left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+k}}$. Eq.(21) implies that firms set the price as a markup over a weighted average of expected future marginal costs. Especially, the first equality in Eq.(22) definitely corresponds to one derived by Gali and Monacelli (2005). The second equality in Eq.(22) is not a familiar expression, although it implies that firms set the price as a markup over a weighted average of expected future nominal marginal costs. The second equality in Eq.(22) is the log-linearized FONC for firms that produce goods in country H and sell them in country F. Those firms set the price in terms of country F's currency as a markup over a weighted average of expected future nominal marginal costs in terms of country F's currency. We establish the character of Eq.(22) after discussing some identities including the relative prices that are peculiar to LCP behavior. Under LCP, the LOOP is not necessarily applied because of Eqs.(20) and (22), which implies that firms set their price of goods in terms of their local currency, namely LCP. Because of that setting, there is the LOOP gap, which measures the degree of the pass-through. Now, we discuss the LOOP gap and the real exchange rate in our model. Following Monacelli (2005), we define the LOOP gap as follows:

$$\Psi_{H,t} \equiv \frac{\mathcal{E}_t P^*_{H,t}}{P_{H,t}} \ ; \ \Psi_{F,t} \equiv \frac{\mathcal{E}_t P^*_{F,t}}{P_{F,t}},$$

where $\Psi_{H,t}$ and $\Psi_{F,t}$ denote the LOOP gap for goods produced in countries H and F, respectively. When the LOOP is applied, we have $\Psi_{H,t} = \Psi_{F,t} = 1$.

Combining Eq.(7), the optimal risk-sharing condition and the definition of the TOT yields:

$$\Psi_{H,t} = \Psi^{-1}_{F,t} \mathcal{S}^{-1}_t \left(\frac{\mathcal{E}_t P^*_{F,t}}{P_{H,t}} \right) \mathcal{Q}_t^{-\frac{1}{\sigma\eta}},$$

which implies that the LOOP gap is a function of the TOT, the real exchange rate and the relative price of goods consumed domestically. Because $\mathcal{S}^{-1}_t \left(\frac{\mathcal{E}_t P^*_{F,t}}{P_{H,t}} \right) = \Psi_{H,t}\Psi_{F,t}$, that equality can be rewritten as follows:

$$\mathcal{Q}_t = 1,$$

which implies that PPP is applied, even if the LOOP is not applied.[6] The log-linearized version of this equality is given by:

$$q_t = 0. \tag{23}$$

In addition, plugging Eq.(23) into Eq.(11), we have $c_t = c^*_t$, which implies that the marginal utilities of consumption in both countries are equal. In fact, households in both countries consume the same goods, although there is price discrimination. As mentioned, the LOOP is not necessarily applied, although Eq.(23) implies that PPP is definitely applied. This may seem inconsistent. However, when the price of some good violates the LOOP, the PPP is applied when another good violates the LOOP inversely. In fact, plugging $\mathcal{Q}_t = 1$ into that equality, we have $\Psi_{H,t} = \Psi^{-1}_{F,t}$ and the log-linearized version of this as

[6]This equality implies that the marginal utility of consumption in country H is definitely the same as in country F. Hence, the UIP can be derived by simply combining Eq.(8) and its counterpart in country F without assuming it, although we mentioned assuming the UIP in Section 2.1.1.

follows:

$$\psi_{H,t} = -\psi_{F,t},$$

which implies that gains from price discrimination correspond to losses from price discrimination.

Log-linearized market clearing conditions in countries H and F clarify the relationships between the nominal exchange rate, the price level and the TOT. Plugging the log-linearized definition of the CPI into log-linearized market clearing conditions yields:

$$\begin{aligned} e_t &= p_t - p_t^* \\ &= p_{P,t} - p_{P,t}^* + s_t, \\ &= p_{P,t} - p_{P,t}^* + \frac{1}{\eta}\left(y_t - y_t^*\right) \end{aligned} \tag{24}$$

where we use Eq.(16) to derive the second line and Eq.(41) to derive the third line. Eq.(24) implies that output differentials between both countries affect the nominal exchange rate and vice versa.

In turn, we discuss the character of Eq.(22), log-linearized FONCs for firms under LCP. By taking the definition of the LOOP gap, Eq.(22) can be rewritten as follows:

$$\begin{aligned} \tilde{p}_{H,t} &= p_{H,t-1} + \sum_{k=0}^{\infty} (\beta\theta)^k \mathrm{E}_t\left(\pi_{H,t+k}\right) + \frac{1-\beta\theta}{2} \sum_{k=0}^{\infty} (\beta\theta)^k \mathrm{E}_t\left(\psi_{H,t+k}\right) \\ &\quad + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathrm{E}_t\left(mc_{t+k}\right), \\ \tilde{p}_{H,t}^* &= p_{H,t-1}^* + \sum_{k=0}^{\infty} (\beta\theta)^k \mathrm{E}_t\left(\pi_{H,t+k}^*\right) - \frac{1-\beta\theta}{2} \sum_{k=0}^{\infty} (\beta\theta)^k \mathrm{E}_t\left(\psi_{H,t+k}\right) \\ &\quad + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathrm{E}_t\left(mc_{t+k}\right), \end{aligned} \tag{25}$$

where $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ and $\pi_{H,t}^* \equiv p_{H,t}^* - p_{H,t-1}^*$ denote inflation of goods both produced and sold in country H and inflation of goods produced in country H and sold in country F, respectively. As mentioned, firms set the price with a markup over a weighted average of future marginal cost. In our LCP setting, those firms' sales are not measured by the PPI, because it is the weighted average of prices of goods selling in both country H and country F. However, real marginal cost is measured by the PPI, as shown in the definition of nominal marginal cost. That is, those firms obtain sales measured by $P_{H,t}$ and pay total costs measured by the $P_{P,t}$, and that gap is calculated by $p_{P,t} - p_{H,t} = \frac{1}{2}\psi_{H,t}$, which implies that the gap corresponds to the LOOP gap in country H. Although the firms selling goods in country H have no currency disparity in sales and payment, LCP behavior generates the LOOP gap. Thus, a weighted average of expected future LOOP gap in country H appears in the first equality in Eq.(25).

The price-setting behavior of the firms selling goods in country F generates the LOOP gap, similar to that of another firm that sells goods in country H. Those firms, namely exporters, obtain the sales of goods exported in terms of country F's currency and pay the total cost in terms of country H's currency. Their sales are measured by country H's currency. Hence, their sales in terms of country F's currency are multiplied by the nominal exchange rate. They pay total costs that are measured by the PPI, similar to that of the firms selling goods in country H. The gap is calculated by $p_{P,t} - \left(p^*_{H,t} + e_t\right) = -\frac{1}{2}\psi_{H,t}$. Thus, a weighted average of the expected future LOOP gap in country H appears in the second equality in Eq.(25), although the sign is contrary to the first equality. Similar mechanisms work in firms in country F, not only for selling goods domestically but also for exporters. Although our LCP setting is different from Monacelli (2005), who assumes a small open economy and importers, our LCP setting clearly generates the LOOP gap, and this setting affects the forms of the NKPC and social welfare stemming from a second-order approximated utility function.

2.1.4 Marginal Cost and Natural Rate of Output

Plugging Eq.(9) into the definition of the marginal cost, we obtain the following:

$$MC_t = (1-\tau)\frac{C_t^{\sigma} N_t^{\varphi}}{A_t}\left(\frac{P_{P,t}}{P_t}\right)^{-1}, \tag{26}$$

which is log-linearized as follows:

$$mc_t = \sigma c_t + \varphi n_t + \frac{1}{2}s_t - a_t, \tag{27}$$

which is consistent with Gali and Monacelli's (2005) log-linearized marginal cost.

Under the flexible price equilibrium, $MC_t = \frac{1}{\zeta}$, implying that the real marginal cost is constant and corresponds to the inverse of a constant markup. Using this fact, and combining Eqs.(14), (18) and Eq.(26), we have the natural rate of output under LCP in country H as follows:

$$\bar{Y}_t = \frac{1}{2}\left\{\frac{P_{P,t}}{P_t}\frac{\zeta^{-1}}{1-\tau}A_t^{1+\varphi}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta\sigma}\left[1+\left(\frac{P_{H,t}}{P_t}\right)^{\eta}\left(\frac{P_{H,t}^{*}}{P_t^{*}}\right)^{-\eta}\mathcal{Q}_t^{-\frac{1}{\sigma}}\right]^{\sigma}\right\}^{\frac{1}{\varphi+\sigma}} D_t^{-\frac{\varphi}{\varphi+\sigma}},$$

where $\bar{Y}_t$ denotes the natural rate of output in country H, which implies that the natural rate of output is a function not only of productivity but also of relative prices because of the open-economy setting.

Before log-linearizing this equality, we define the output gap in country H x_t as the percentage deviation of output in country H y_t from its natural level $\bar{y}_t$. This relationship can be written as:

$$x_t \equiv y_t - \bar{y}_t, \tag{28}$$

which is definitely consistent with Gali and Monacelli's (2005) definition. The output gap in country F is defined analogously to Eq.(28).

Now, we log-linearize that equality. The log-linearized natural rate of output under LCP is given by:

$$\bar{y}_t = \frac{\omega_1\omega_2}{\omega_3}a_t - \frac{(\sigma\eta-1)\,\omega_2}{\omega_3}a_t^{*}, \tag{29}$$

with $\omega_1 \equiv \eta(\sigma + 2\varphi) + 1$, $\omega_2 \equiv 2\eta(1+\varphi)$ and $\omega_3 \equiv \omega_1^2 - (\sigma\eta - 1)^2$. While Gali and Monacelli (2005) regard foreign output as exogenous because of their small open-economy setting, foreign output, namely output in country F is endogenous in our two-country setting. Thus, productivity in country F replaces foreign output in Eq.(29).

We turn to discuss Eq.(27), percentage deviation of marginal cost from its steady state value. Plugging Eqs.(19), (28), (41) and (42), into Eq.(27) yields:

$$mc_t = \frac{\omega_1}{2\eta} x_t + \frac{\sigma\eta - 1}{2\eta} x_t^*, \tag{30}$$

which implies that real marginal cost in country H consists of the output gap in both countries.

2.1.5 The Demand and Supply Sides

Plugging Eqs.(21), (23),(41) and (42) into Eq.(10) yields the New Keynesian IS Curve (NKIS) as follows:

$$x_t = \mathrm{E}_t(x_{t+1}) - \frac{2\eta}{\sigma_\alpha}\hat{r}_t + \frac{2\eta}{\sigma_\alpha}\mathrm{E}_t(\pi_{P,t+1}) + \frac{\sigma\eta - 1}{\sigma_\alpha}\mathrm{E}_t\left(\Delta x_{t+1}^*\right) + \frac{2\eta}{\sigma_\alpha}\bar{r}_t, \tag{31}$$

where $\bar{r}_t \equiv -\sigma_\alpha \frac{(1-\rho)(1+\varphi)\omega_4}{\omega_3} a_t - \sigma_\alpha \frac{(1-\rho)(\sigma\eta-1)(1+\varphi)\omega_5}{\omega_3} a_t^*$ denotes the natural rate of interest in country H with $\sigma_\alpha \equiv \sigma\eta + 1$, $\omega_4 \equiv \omega_1 - \frac{(\sigma\eta-1)^2}{\sigma_\alpha}$ and $\omega_5 \equiv \frac{\omega_1}{\sigma_\alpha} - 1$. The NKIS in country F, which is analogous to Eq.(31), and can be derived by using Eqs.(23) and (41) and counterparts of Eqs.(10), (21) and (42).

Eq.(31) looks like the ordinary NKIS in the DSGE literature at a glance. Because of LCP, however, Eq.(31) has some distinguishing features. Plugging Eq.(21) into Eq.(31), the NKIS under LCP can be rewritten as follows:

$$\begin{aligned} x_t &= \mathrm{E}_t(x_{t+1}) - \frac{2\eta}{\sigma_\alpha}\hat{r}_t + \frac{\eta}{\sigma_\alpha}\mathrm{E}_t(\pi_{H,t+1}) + \frac{\eta}{\sigma_\alpha}\mathrm{E}_t\left(\pi_{H,t+1}^*\right) + \frac{\eta}{\sigma_\alpha}\mathrm{E}_t(\Delta e_{t+1}) \\ &\quad + \frac{\sigma\eta - 1}{\sigma_\alpha}\mathrm{E}_t\left(\Delta x_{t+1}^*\right) + \frac{2\eta}{\sigma_\alpha}\bar{r}_t \\ &= \mathrm{E}_t(x_{t+1}) - \frac{\eta}{\sigma_\alpha}\hat{r}_t - \frac{\eta}{\sigma_\alpha}\hat{r}_t^* + \frac{\eta}{\sigma_\alpha}\mathrm{E}_t(\pi_{H,t+1}) + \frac{\eta}{\sigma_\alpha}\mathrm{E}_t\left(\pi_{H,t+1}^*\right) \\ &\quad + \frac{\sigma\eta - 1}{\sigma_\alpha}\mathrm{E}_t\left(\Delta x_{t+1}^*\right) + \frac{2\eta}{\sigma_\alpha}\bar{r}_t, \end{aligned} \tag{32}$$

where we take log-linearized UIP to derive the second line. As shown in the first line, changes in expected nominal exchange rates affect the NKIS. The second line shows that not only the domestic nominal interest rate, but also the foreign nominal interest rate appears in the NKIS.

Plugging the log-linearized Calvo's pricing rule and Eq.(30) into Eq.(25), we have equalities that determine the dynamics of inflation as follows:

$$\begin{aligned} \pi_{H,t} &= \beta \mathrm{E}_t\left(\pi_{H,t+1}\right) + \frac{\lambda}{2}\psi_{H,t} + \frac{\lambda\omega_1}{2\eta}x_t + \frac{\lambda\left(\sigma\eta-1\right)}{2\eta}x_t^*, \\ \pi_{H,t}^* &= \beta \mathrm{E}_t\left(\pi_{H,t+1}^*\right) - \frac{\lambda}{2}\psi_{H,t} + \frac{\lambda\omega_1}{2\eta}x_t + \frac{\lambda\left(\sigma\eta-1\right)}{2\eta}x_t^*, \end{aligned} \tag{33}$$

with $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$. The first equality is inflation dynamics for goods sold domestically, and the second equality is inflation dynamics for goods exported. Because Eq.(33) is derived from Eq.(25), the FONCs for firms in country H, the third and the fourth terms in the RHS, which stem from real marginal cost in country H, are consistent between both equalities. The signs of the second terms in the RHS are inverse between both equalities. The reason is that the losses from price discrimination are compensated by the gains from price discrimination, and vice versa. The counter part of Eq.(33) is derived from the counter part of Eq.(25).

Plugging Eq.(33) into Eq.(21), we have the NKPC in country H as follows:

$$\pi_{P,t} = \beta \mathrm{E}_t\left(\pi_{P,t+1}\right) + \frac{\lambda\omega_1}{2\eta}x_t + \frac{\lambda\left(\sigma\eta-1\right)}{2\eta}x_t^* - \frac{\beta}{2}\mathrm{E}_t\left(\Delta e_{t+1}\right) + \frac{1}{2}\Delta e_t,$$

and plugging the counter part of Eq.(33) into the counterpart of Eq.(21) yields the counter part of this equality in country F. This NKPC features the appearance of changes in the nominal exchange rate. Gali and Monacelli (2005) mention that full stabilization of domestic prices coincides with full stabilization of the output gap, namely $x_t = \pi_{H,t} = 0$ for all t. In our model, their domestic prices correspond to the PPI and they assume fully exogenous foreign output, which implies that the percentage deviation of marginal cost from its steady state value is not affected by the percentage deviation of foreign output from its steady state value. That is, they claim that full stabilization of the PPI implies that output conforms to its natural rate if we ignore the foreign output gap or assume $\sigma\eta = 1$ in this equality. Even if we ignore the foreign output gap or assume $\sigma\eta = 1$ in this equality, full stabilization of the PPI does not necessarily imply that output conforms to its natural rate because of changes in nominal exchange rate, as shown in the fourth and fifth terms in the RHS. Changes in the nominal exchange rate resemble a cost push shock in the NKPC under LCP. Thus, full stabilization of the PPI no longer implies that output conforms to its natural rate if we ignore the foreign output gap or assume $\sigma\eta = 1$. Plugging Eqs.(24), (28) and (29) into that equality, we can eliminate changes in the nominal exchange rate and obtain the following:

$$\begin{aligned}\pi_{P,t} \quad = \quad & \beta \mathrm{E}_t\left(\pi_{P,t+1}\right) + \beta \mathrm{E}_t\left(\pi^*_{P,t+1}\right) - \frac{\beta}{\eta}\mathrm{E}_t\left(x_{t+1}\right) + \frac{\beta}{\eta}\mathrm{E}_t\left(x^*_{t+1}\right) + \kappa_{\varpi} x_t \\ & - \kappa_{\varsigma} x^*_t - \pi^*_{P,t} - \frac{1}{\eta}x_{t-1} + \frac{1}{\eta}x^*_{t-1} + \omega_6 a_t - \omega_6 a_{t-1} - \omega_6 a^*_t + \omega_6 a^*_{t-1},\end{aligned} \tag{34}$$

with $\kappa_{\varpi} \equiv \frac{1+\beta+\lambda\omega_1}{\eta}$, $\kappa_{\varsigma} \equiv \frac{1+\beta-\lambda(\sigma\eta-1)}{\eta}$, $\omega_6 \equiv \frac{\omega_2\varpi_3(\sigma+\varphi)}{\omega_3}$ and $\varpi_3 \equiv 1+\beta\left(1-\rho\right)$. Exogenous shocks appear in Eq.(34), which shows that exogenous productivity affects PPI inflation.

Monacelli (2005) derives CPI-based NKPC. Following Monacelli (2005), we derive CPI-based NKPC. Plugging the first equality in Eq.(33) and its counterpart in country F into Eq.(5) yields:

$$\pi_t = \beta \mathrm{E}_t\left(\pi_{t+1}\right) + \frac{\kappa_\alpha}{2}x_t + \frac{\kappa_\alpha}{2}x^*_t, \tag{35}$$

with $\kappa_\alpha \equiv \lambda\left(\sigma+\varphi\right)$. As mentioned by Gali and Monacelli (2005), κ_α is consistent with the slope coefficient of a standard closed economy NKPC. A full stabilization, not of PPI inflation but of CPI inflation, implies that output conforms to its natural rate when the nominal interest rate in both countries absorbs the effects from changes in productivity in the NKIS. Gali and Monacelli (2005) mention that a full stabilization of PPI inflation implies that output conforms to its natural level and there is no output gap in their non-LCP setting under a small open economy, as mentioned. However, our CPI-based NKPC Eq.(35) implies that a full stabilization of CPI inflation implies that output conforms to its natural level and there is no output gap in our LCP setting under a two-country economy. This can be understood alternatively and intuitively by comparing Eqs.(5) and (21). To derive Eq.(34), we use Eq.(21), implying that PPI inflation is affected by changes in the nominal exchange rate, while we use Eq.(5) to derive Eq.(35).

In addition, Eq.(35) contrasts CPI-based NKPC in Monacelli (2005). In his LCP setting, importers purchase foreign goods at costs in terms of foreign currency, while they sell foreign goods at prices in terms of domestic currency. Because importers maximize their profits, the LOOP gap appears in the CPI-based NKPC in Monacelli (2005). Our LCP setting is quite different from Monacelli's (2005) setting. Goods markets are fully partitioned, there are no importers, and each producer prices their goods in terms of the consumer's currency. As mentioned in Section 2.1.3, $\psi_{H,t} = -\psi_{F,t}$ is applied in our LCP model. Gains from price discrimination correspond to losses from price discrimination. Hence, the LOOP gap does not appear in Eq.(35), which is different from Monacelli (2005).

2.2 PCP Model

Under PCP, the LOOP is applied, which is given by $P_t(h) = \mathcal{E}_t P_t^*(h)$ and $P_t(f) = \mathcal{E}_t P_t^*(f)$, hence:

$$P_{H,t} = \mathcal{E}_t P_{H,t}^* \; ; \; P_{F,t} = \mathcal{E}_t P_{F,t}^* \tag{36}$$

and

$$p_{H,t} = e_t + p^*_{H,t} \ ; \ p_{F,t} = e_t + p^*_{F,t} \tag{37}$$

are applied.

2.2.1 Households

The preference of the representative household, private consumption index, consumption index, the optimal allocation of any given expenditure within each category of goods and the optimal allocation of expenditures between domestic and foreign goods are given by Eqs.(1), (2), (3), (4), (6) and (7), as with the LCP model. Because households face the same optimization problem, intertemporal and intratemporal optimality conditions are given by Eqs.(8) and (9). The UIP is applied in the PCP model, hence the optimal risk-sharing condition is applied in the PCP model. The log-linearized definition of the CPI, the intertemporal optimality condition and the risk-sharing condition are also given by Eqs.(5), (10) and (11).

2.2.2 Market Clearing

The market clearing condition is given by Eq.(12), as with the LCP model. Plugging Eq.(6) into Eq.(12), we have Eq.(13). Because of LOOP, Eq.(13) can be rewritten as:

$$Y_t(h) = \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t$$

by utilizing Eq.(36). Plugging the Dixit–Stiglitz aggregator of output into this equality yields:

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t, \tag{38}$$

which is a demand function consistent with that of Benigno and Benigno (2006).[7]

[7] We do not assume government expenditure. Thus, government expenditure does not appear in these equalities, although it appears in Benigno and Benigno (2006).

A definition of the TOT is given by Eq.(15). Plugging Eq.(36) into Eq.(15) yields:

$$\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}}, \tag{39}$$

which is only applicable to the PCP model because Eq.(36) is not applicable to the LCP model.

Log-linearizing Eq.(39), we have:

$$s_t = p_{F,t} - p_{H,t}. \tag{40}$$

Eq.(40) is only applicable to the PCP model because Eq.(37) is not applied under LCP, as with Eq.(39).

Plugging Eq.(37) into this equality, we have:

$$s_t = \frac{1}{\eta}\left(y_t - y_t^*\right), \tag{41}$$

which clarifies the relationship between the TOT and relative output under PCP. Gali and Monacelli (2005) and Benigno and Benigno (2006), who assume PCP, derive the same equality.

Log-linearizing Eq.(38) yields:

$$y_t = \frac{\eta}{2} s_t + c_t, \tag{42}$$

where we use Eq.(37). As mentioned, Eq.(42) is the final form of log-linearized market clearing under LCP in country H. The difference in price-setting behavior between LCP and PCP does not affect market clearing.

2.2.3 Firms

Firms' technology is given by Eq.(17), which can be rewritten as Eq.(18). Thus, log-linearized technology is given by Eq.(19) as with the LCP model.

We assume Calvo–Yun-style price-setting behavior, as with the LCP model. However, the maximization problem that is faced by firms under PCP is quite simple. Because of $\tilde{P}_{H,t} = \mathcal{E}_t \tilde{P}^*_{H,t}$ and Eq.(36), Eq.(20) can be rewritten as:

$$\max_{\tilde{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathrm{E}_t \left\{ Q_{t,t+k} \left[\left(\tilde{P}_{H,t} - MC^n_{t+k} \right) \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left(C_{H,t+k} + C^*_{H,t+k} \right) \right] \right\}, \quad (43)$$

which is a familiar expression in literature assuming Calvo pricing. Plugging Eq.(36) into the PPI definition, we have $P_{P,t} = P_{H,t}$, and plugging Eq.(37) into Eq.(21) yields:

$$\pi_{P,t} = \pi_{H,t}. \quad (44)$$

The FONC of Eq.(43) is given by:

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\tilde{P}_{H,t} - \zeta MC^n_{t+k} \right) \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left(C_{H,t+k} + C^*_{H,t+k} \right) \right] = 0,$$

which is a familiar expression in literature assuming PCP. Log-linearizing this equality, we have:

$$\tilde{p}_{H,t} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathrm{E}_t \left(mc^n_{t+k} \right),$$

which corresponds to the first equality in Eq.(22). Terms related to the LOOP gap disappear because the LOOP is definitely applied in the PCP model. This equality can be rewritten as follows:

$$\tilde{p}_{H,t} = p_{H,t-1} + \sum_{k=0}^{\infty} (\beta\theta)^k \mathrm{E}_t \left(\pi_{H,t+k} \right) + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathrm{E}_t \left(mc_{t+k} \right), \quad (45)$$

which corresponds to one derived by Gali and Monacelli (2005). Because of LOOP, the LOOP gap disappears in Eq.(45), although it does appear in the first equality in Eq.(25).

2.2.4 Marginal Cost and Natural Rate of Output

Plugging Eq.(9) into the definition of the marginal cost, we obtain Eq.(26) and its log-linearized equality Eq.(27). Combining not only Eqs.(14), (18) and Eq.(26) but also $P_{P,t} = P_{H,t}$, we have:

$$\bar{Y}_t = \left[\frac{\zeta^{-1}}{1 - \tau} A_t^{1+\varphi} \left(\frac{P_{H,t}}{P_t} \right)^{-(\sigma\eta - 1)} \right]^{\frac{1}{\sigma+\varphi}} D_t^{-\frac{\varphi}{\sigma+\varphi}},$$

which can be log-linearized as follows:

$$\bar{y}_t = \frac{\omega_1 \omega_2}{\omega_3} a_t - \frac{(\sigma\eta - 1)\,\omega_2}{\omega_3} a_t^*.$$

This equality is consistent with the log-linearized natural rate of output under the LCP Eq.(29), although the natural rate of output is quite different between PCP and LCP before log-linearizing. This implies that differences in price-setting behavior do not affect the natural rate of output.

That the natural rate of output under PCP is consistent with that under LCP implies that there is the same relationship between the marginal cost and the output gap even under PCP. In fact, plugging Eqs.(19), (28), (41) and (42) into Eq.(27) yields:

$$mc_t = \frac{\omega_1}{2\eta} x_t + \frac{\sigma\eta - 1}{2\eta} x_t^*,$$

which is consistent with Eq.(30). The difference between the PCP and the LCP models is their price-setting behavior. Because marginal cost has no relationship with price-setting behavior, Eq.(30) is applied under both PCP and LCP. Note that Gali and Monacelli (2005) show that real marginal cost has a relationship with the domestic output gap, and their result is different from Eq.(30). This difference stems from our two-country setting. As mentioned, foreign output is not exogenous in our setting, and productivity in country F appears in Eq.(29), while foreign output appears in their expression in terms of percentage deviation from its steady state value. In their setting, foreign output, not foreign productivity, affects the natural rate of domestic output. The foreign output gap no longer affects the domestic output gap, which stems from the percentage deviation of domestic real marginal cost from its steady state value. Because the percentage deviation of domestic real marginal cost from its steady state value corresponds to its deviation from its flexible price equilibrium value, the foreign output gap disappears in Gali and Monacelli (2005). In fact, we have $mc_t = \frac{\omega_1}{2\eta} x_t$ if we regard output in country F as exogenous.

2.2.5 The Demand and Supply Sides

Plugging Eqs.(21), (23), (41) and (42) into Eq.(10) yields NKIS as follows:

$$x_t = \mathrm{E}_t\left(x_{t+1}\right) - \frac{2\eta}{\sigma_\alpha}\hat{r}_t + \frac{2\eta}{\sigma_\alpha}\mathrm{E}_t\left(\pi_{P,t+1}\right) + \frac{\sigma\eta-1}{\sigma_\alpha}\mathrm{E}_t\left(\Delta x^*_{t+1}\right) + \frac{2\eta}{\sigma_\alpha}\bar{r}_t, \qquad (46)$$

which is consistent with NKIS under the LCP Eq.(31). While the LOOP is not applied in the LCP model, it is applied in the PCP model. Hence, the NKIS is not the same in both models, although it looks similar. Plugging Eq.(44) into Eq.(31), we have:

$$x_t = \mathrm{E}_t\left(x_{t+1}\right) - \frac{2\eta}{\sigma_\alpha}\hat{r}_t + \frac{2\eta}{\sigma_\alpha}\mathrm{E}_t\left(\pi_{H,t+1}\right) + \frac{\sigma\eta-1}{\sigma_\alpha}\mathrm{E}_t\left(\Delta x^*_{t+1}\right) + \frac{2\eta}{\sigma_\alpha}\bar{r}_t,$$

which is only applicable to the PCP model, and $\pi_{H,t}$ replaces $\pi_{P,t}$ in this equality. Because LOOP definitely applies in the PCP model, neither changes in expected nominal exchange rate nor foreign nominal interest rates appear in the NKIS under PCP.

By rearranging Eq.(45), we have NKPC in country H under PCP as follows:

$$\pi_{P,t} = \beta\mathrm{E}_t\left(\pi_{P,t+1}\right) + \frac{\lambda\omega_1}{2\eta}x_t + \frac{\lambda\left(\sigma\eta-1\right)}{2\eta}x^*_t, \qquad (47)$$

which is the two-country version of the NKPC derived by Gali and Monacelli (2005). While a foreign output gap appears in Eq.(47), it does not appear in the NKPC derived by Gali and Monacelli (2005), who assume a small open economy where foreign variables are exogenous. Because our model is a two-country model where the foreign variables are endogenous, a foreign output gap appears in our NKPC under PCP. In fact, if we regard output in country F as exogenous, we have:

$$\pi_{P,t} = \beta\mathrm{E}_t\left(\pi_{P,t+1}\right) + \frac{\lambda\omega_1}{2\eta}x_t,$$

which is quite similar to the NKPC derived by Gali and Monacelli (2005) and can be derived alternatively only if $\sigma\eta = 1$ in our two-country model under PCP because the foreign output gap disappears in such a case. Gali and Monacelli (2005) mention that full stabilization of the PPI implies that $x_t = \pi_{H,t} = 0$, which is plausible if the output gap in country F disappears in Eq.(47). Because of the two-country setting, the foreign output gap does not disappear as long as we do not assume $\sigma\eta = 1$. Hence, full stabilization of the PPI does not necessarily imply $x_t = \pi_{H,t} = 0$ in our two-country setting.

3 Optimal Monetary Policy under LCP and PCP

3.1 Welfare Costs

We assume central banks adopt an optimal monetary policy, and that they seek to minimize welfare costs. Welfare costs consist of the period loss function derived from the welfare criterion. Following Woodford (2003) and Gali (2008), we have this second-order approximated utility function:

$$\mathcal{W}^W_{LCP} = -\mathcal{L}^W_{LCP} + \text{t.i.p.} + o\left(\| \xi \|^3\right) \ ; \ \mathcal{W}^W_{PCP} = -\mathcal{L}^W_{PCP} + \text{t.i.p.} + o\left(\| \xi \|^3\right), \quad (48)$$

where $\mathcal{L}^W_{LCP} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t L^W_{LCP,t}$ and $\mathcal{L}^W_{PCP} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t L^W_{PCP,t}$ denote the loss function in the LCP and PCP models, respectively, $\mathcal{W}^W_{LCP} = \frac{1}{2}\left(\mathcal{W}_{LCP} + \mathcal{W}^*_{LCP}\right)$ and $\mathcal{W}^W_{PCP} = \frac{1}{2}\left(\mathcal{W}_{PCP} + \mathcal{W}^*_{PCP}\right)$ denote average welfare criteria in the LCP and PCP models, respectively, $\mathcal{W}_{LCP}$ and $\mathcal{W}_{PCP}$ denote welfare criteria in country H in the LCP and PCP models, respectively, with $\mathcal{W} \equiv \sum_{t=0}^{\infty} \mathrm{E}_0\left(\mathcal{W}_t\right)$ and $\mathcal{W}_t \equiv \frac{U_t - U}{U_C C}$. Further:

$$L^W_{LCP,t} \equiv \frac{1}{2}\left[\frac{\varepsilon}{2\lambda}\pi_t^2 + \frac{\varepsilon}{2\lambda}\left(\pi_t^*\right)^2 + (\sigma+\varphi)\left(x_t^W\right)^2 + \frac{(1+\varphi)\eta^2}{4} z_t^2\right], \quad (49)$$

$$L^W_{PCP} \equiv \frac{1}{2}\left[\frac{\varepsilon}{2\lambda}\pi_{H,t}^2 + \frac{\varepsilon}{2\lambda}\left(\pi_{F,t}^*\right)^2 + (\sigma+\varphi)\left(x_t^W\right)^2 + \frac{(1+\varphi)\eta^2}{4} z_t^2\right], \quad (50)$$

are the period loss function in countries H and F, respectively, $z_t \equiv s_t - \bar{s}_t$ being the deviation of the TOT from its efficient level, and $\bar{s}_t \equiv \frac{1+\varphi\eta}{\eta^2(1+\varphi)}\bar{y}_t^R$ being the efficient level of the TOT. Note that we define $v_t^W \equiv \frac{1}{2}\left(v_t + v_t^*\right)$ and $v_t^R \equiv v_t - v_t^*$.

3.2 FONCs for Central Banks

Next, we briefly mention the FONCs for the central banks. We assume that the central bank in each country cooperatively conducts optimal monetary policy with commitment. Under LCP, central banks minimize Eq.(49) and the FONCs for them are given by:

$$\pi_t^W = -\frac{1}{\varepsilon}\left(x_t^W - x_{t-1}^W\right), \tag{51}$$

$$z_t = 0, \tag{52}$$

where $\mu_{3,t}$ is a Lagrange multiplier associated with the average block of the NKPC. Because of commitment, lagged variables appear in the FONCs. Eqs.(51) and (52) determine the equilibrium path of the output gap, CPI inflation and the deviation of the TOT from its efficient level in LCP along with the structural model. Eq.(51) implies that there are no trade-offs between output gap and inflation in a total economy consisting of two countries under optimal monetary policies. This implication is consistent with outcomes under the assumption of a closed economy. Eq.(52) implies that fully stabilizing the deviation of the TOT from its efficient level is optimal regardless of preferences such as the elasticity of substitution between goods produced in countries H and F η and the relative risk aversion σ.

Under PCP, central banks minimize Eq.(50) and the FONCs for them are given by:

$$\begin{aligned} \pi_t^W &= -\frac{1}{\varepsilon}\left(x_t^W - x_{t-1}^W\right), \\ \pi_{P,t}^R &= -\frac{(1+\varphi)\,\eta^2}{\varepsilon\,(1+\eta\varphi)}\left(z_t - z_{t-1}\right). \end{aligned} \tag{53}$$

One of the FONCs is consistent with Eq.(51), and Eqs.(51) and (53) determine the equilibrium path of the output gap, PPI inflation and the deviation of the TOT from its efficient level in PCP along with the structural model. Because one of the FONCs is consistent with Eq.(51), there are no trade-offs between output gap and inflation in the total economy consisting of two countries, under optimal monetary policy. Eq.(53) does not show clear implications on trade-offs between the output gap and inflation. However, under the special case, we can understand what Eq.(53) implies. By plugging $\eta = 1$ into Eq.(53), we have:

$$\pi_{P,t}^{R} = -\frac{1}{\varepsilon}\left(x_t^R - x_{t-1}^R\right),$$

which implies that the inflation–output gap trade-offs are fully dissolved in each country with optimal monetary policy when the elasticity of substitution between goods produced in countries H and F is unity.

3.3 Calibration

We run a series of dynamic simulations and adopt the following benchmark parameterization. We set price stickiness θ, the subjective discount factor β, the elasticity of substitution across goods ε and the inverse of the labor supply elasticity φ equal to 0.75, 11, 0.99 and 3, respectively, consistent with the quarterly time periods in the model.[8] We compare two cases. One of them is the special case in which $\sigma = \eta = 1$, and the other is the general case in which $\sigma = 3$ and $\eta = 4.5$. Here, σ is the relative risk aversion and η is the elasticity of substitution between goods produced in countries H and F. Note that $\sigma = \eta = 1$ is assumed by Gali and Monacelli (2005), while $\sigma = 3$ and $\eta = 4.5$ are assumed by Benigno and Benigno (2006). We assume that productivity shifters are described according to the following AR(1) processes:

$$a_t = \rho a_{t-1} + \xi_t \ ; \ a_t^* = \rho a_{t-1}^* + \xi_t^*,$$

where ξ_t and ξ_t^* denote the i.i.d. shocks. We set ρ equal to 0.9. To examine the impulse response functions (IRFs), we consider one-percent increases in the productivity shifter in country H, a_t, and the productivity shifter in country F, a_t^*.

[8] $\theta = 0.75$ implies an average length of price contracts equal to 4.

Impulse responses to one-percent increases in productivity in country H in the special case are shown in Figure 1, while macroeconomic volatilities are shown in the 3rd and 4th columns in Table 1[9]. To eliminate the effects of changes in productivity, central banks decrease nominal interest rates under both LCP and PCP (Panels 7 and 8). Because of this, the output gaps in countries H and F are completely stabilized (Panels 1 and 2). Under PCP, the PPI inflation rate in countries H and F is completely stabilized (Panels 3 and 4). This result is consistent with Gali and Monacelli (2005), who imply that PPI inflation targeting yields a zero output gap. This result can be understood by paying attention to Eq.(47). Plugging $\sigma = \eta = 1$ into Eq.(47) yields:

$$\pi_{P,t} = \beta \mathrm{E}_t \left(\pi_{P,t+1} \right) + \frac{\lambda \omega_1}{2\eta} x_t,$$

which is the NKPC in the special case under PCP. This NKPC implies that stabilizing PPI inflation also simultaneously stabilizes the output gap, and is consistent with the result derived by Gali and Monacelli (2005), although the slope of our NKPC is slightly different from theirs because we assume a two-country economy.

Under LCP, it is CPI, not PPI inflation that is stabilized, and this result is quite different not only from Gali and Monacelli (2005) but also from other DSGE papers assuming an open economy (Panels 5 and 6). This can be understood by paying attention to Eq.(35), which implies that CPI inflation becomes zero when the output gaps in countries H and F are stabilized. Hence, it may be said that CPI inflation targeting brings complete stability to the output gap. Interestingly, the nominal exchange rate is completely stabilized under LCP, which is consistent with Devereux and Engel (2003) developing their New Open Economy Macroeconomics model, assuming LCP and showing that a fixed exchange rate is the optimal regime for maximizing welfare. This result stems

[9] There are 2 eigenvalues larger than 1 in modulus for forward-looking variables. Hence, the rank condition is verified

from stabilizing the CPI inflation rate. Perfect stabilization in CPI inflation is consistent with perfect stabilization in the CPI level.[10] In our model, PPP is always applied, hence $e_t + p_t^* = p_t$ is applied. Perfect stabilization in CPI inflation implies $p_t = p_t^* = 0$, which is consistent with $e_t = 0$. Thus, under LCP there are no changes in either CPI inflation or the nominal exchange rate.

Impulse responses to a one-percent increase in productivity in country H in the general case are shown in Figure 2.[11] In the general case under PCP, inflation–output gap trade-offs are no longer dissolved simultaneously, although Gali and Monacelli (2005) show that trade-offs are dissolved simultaneously (Panels 1 to 4). Because ours is a two-country model, foreign output is endogenous, while it is exogenous in Gali and Monacelli's (2005) small open-economy model. In a small economy setting, the foreign output gap disappears in the NKIS, although it appears in our NKIS, as shown in Eq.(47). The foreign output gap disappears in Eq.(47) only if $\sigma\eta = 1$ in our two-country setting. Hence, neither the output gap nor PPI inflation are stabilized simultaneously.

However, although the output gap in countries H and F is not stabilized, CPI inflation is completely stabilized under LCP. This can be understood by paying attention to Eq.(35). The average output gap is always stabilized, not only under PCP but also under LCP (Rows 3 and 4 in Table 1). Hence, CPI inflation is stabilized because Eq.(35) can be rewritten as:

$$\pi_t = \beta \mathrm{E}_t \left(\pi_{t+1} \right) + \kappa_\alpha x_t^W.$$

In this NKPC, the slope is not affected by σ and η. Thus, the result in terms of volatility is similar in the special and the general cases. In addition, CPI inflation is completely stabilized, and there is no fluctuation in the nominal exchange rate, as with the special case (Panel 13). As mentioned, our result that there is no fluctuation in the nominal exchange rate under LCP is consistent

[10] We assume zero inflation deterministic steady state.

[11] There are two eigenvalues larger than 1 in modulus for two forward-looking variables. Hence, the rank condition is verified.

with the result of Devereux and Engel (2003). Devereux and Engel (2003) assume the Armington form of consumption, which is consistent with $\eta = 1$ in our model.[12] Now, we apply a more general setting, such as $\eta = 4$, while our result on fluctuations in the nominal exchange rate is consistent with their result. This implies that Devereux and Engel's (2003)finding can be applied in general parameterization. We discuss this topic further in the next section.

4 Macroeconomic Volatilities and Welfare Costs

In this section, we focus on macroeconomic volatilities and welfare costs when varying the relative risk aversion σ and the elasticity of substitution between goods produced in countries H and F η. There are many macroeconomic variables in our model, and we focus on some important variables that are related to our loss functions Eqs.(49) and (50) and the nominal exchange rate.

Figure 3 shows effects on macroeconomic volatilities of varying the relative risk aversion σ from 1 to 10 and the elasticity of substitution between goods produced in countries H and F η from 1 to 10. Under PCP, volatility of PPI inflation is definitely zero when $\eta = 1$, although the higher the η the higher the volatility (Panel 2). When $\eta = 1$, one of the FONCs related to relative inflation for central banks under PCP can be rewritten as:

$$\pi^R_{P,t} = -\frac{1}{\varepsilon}\left(x^R_t - x^R_{t-1}\right) \tag{54}$$

because $z_t = x^R_t$ is applied when $\eta = 1$. Along with another FONC related to the average output gap for central banks under PCP, those FONCs imply that stabilization in PPI inflation is strictly consistent with stabilization in the output gap. Hence, volatility of PPI inflation is definitely zero when $\eta = 1$. Equally, this implies that $\sigma = \eta = 1$ is not a sufficient condition to dissolve inflation–output gap trade-offs, but $\eta = 1$ is a sufficient condition to dissolve

[12] In that case, Eq.(2) is rewritten by $C_t = 2C^{\frac{1}{2}}_{H,t}C^{\frac{1}{2}}_{F,t}$.

those trade-offs under PCP. As implied by calibration in the former section, the volatility of CPI inflation is definitely zero regardless of σ and η under LCP (Panel 3). This stems from the NKIS under LCP Eq.(35). Because of FONCs for central banks related to average inflation, there is no fluctuation in average output gap regardless of σ and η. This immediately implies that there is no fluctuation in CPI inflation in countries H and F regardless of σ and η under LCP.

The average output gap is completely stabilized under both LCP and PCP regardless of σ and η (Panels 5 and 6). This stems from the FONC for central banks related to average inflation, which are common under both LCP and PCP, and imply that average inflation and the output gap are stabilized simultaneously. TOT deviation from the efficient level is definitely zero regardless of σ and η under LCP (Panel 7). This stems from the FONC for central banks under LCP related to TOT deviation from the efficient level. However, TOT deviation from the efficient level is definitely zero under PCP only if $\eta = 1$ (Panel 8). In that case, $z_t = x_t^R$ is applied and there is no fluctuation in output gap in countries H and F, as implied by Eq.(54). Both PPI inflation and the output gap are completely stabilized when $\eta = 1$. Hence, TOT deviation from the efficient level is definitely zero through complete stabilization in the output gap in countries H and F. However, complete stabilization in TOT deviation from the efficient level is no longer achieved when $\eta = 1$ is not applied.

Now, we discuss volatility of the nominal exchange rate. As many authors have shown, we also find that optimal monetary policy is consistent with a flexible exchange rate regime (Panel 10) under PCP. On the contrary, the nominal exchange rate is definitely zero regardless of σ and η under LCP. Even if LCP is assumed, PPP is applied, which implies that $s_t = p_t - p_t^*$. Because of optimal monetary policy, CPI inflation is definitely stabilized, which is consistent with zero fluctuation in the CPI level under LCP. Thus, the nominal exchange rate is definitely stabilized regardless of σ and η under LCP. As mentioned in the former section, our result is consistent with Devereux and Engel's (2003)result, which shows that optimal monetary policy under LCP is consistent with a fixed exchange rate regime, although they assume an Armington form of consumption that corresponds to $\eta = 1$ in our model. Our model does not assume the Armington form of consumption, and there is no parametric restriction in η. Thus, our result implies that Devereux and Engel's (2003) policy implication is not only applicable with special parameterization, but also applicable in a general setting. In addition, we derive more important policy implications. Optimal monetary policy under LCP definitely stabilizes CPI inflation and coincides with complete stabilization of the nominal exchange rate. Furthermore, monetary policy that stabilizes CPI inflation or the nominal exchange rate is optimal under LCP. We do not analyze an explicit targeting rule or a regime such as CPI inflation targeting and a fixed exchange rate regime. However, it can be said that CPI inflation targeting and a fixed exchange rate regime are optimal and equivalent under LCP, although there is some room to discuss this more precisely.

Finally, we discuss the effects on welfare costs of varying the relative risk aversion σ and the elasticity of substitution between goods produced in countries H and F η. When we introduce $\beta \rightarrow 1$ in Eq.(48), we have welfare criteria as follows:

$$\begin{aligned}
\mathcal{L}^W_{LCP,t} &\equiv \frac{1}{2}\left[\frac{\varepsilon}{2\lambda}\text{var}\left(\pi_t\right) + \frac{\varepsilon}{2\lambda}\text{var}\left(\pi^*_t\right) + \left(\sigma+\varphi\right)\text{var}\left(x^W_t\right) + \frac{\left(1+\varphi\right)\eta^2}{4}\text{var}\left(z_t\right)\right], \\
\mathcal{L}^W_{PCP,t} &\equiv \frac{1}{2}\left[\frac{\varepsilon}{2\lambda}\text{var}\left(\pi_{P,t}\right) + \frac{\varepsilon}{2\lambda}\text{var}\left(\pi^*_{P,t}\right) + \left(\sigma+\varphi\right)\text{var}\left(x^W_t\right) + \frac{\left(1+\varphi\right)\eta^2}{4}\text{var}\left(z_t\right)\right],
\end{aligned}$$

and we utilize these equalities to calculate welfare costs.

Figure 4 depicts the effects on welfare costs of varying the relative risk aversion σ and the elasticity of substitution between goods produced in countries H and F η. We have already discussed some macroeconomic volatilities that produce welfare costs. Thus, we can understand effects on welfare costs of varying σ and η. Because there are no fluctuations in the average output gap, CPI inflation in countries H and F and the TOT deviation from the efficient level regardless of σ and η, there are no welfare costs regardless of σ and η under LCP. However, there are no welfare costs only if $\eta = 1$, because there are no fluctuations in PPI inflation in countries H and F and the TOT deviation from the efficient level under PCP. Except for $\eta = 1$, there are some welfare costs because there are fluctuations in PPI inflation in countries H and F and the TOT deviation from the efficient level under PCP, even if optimal monetary policy is conducted.

Because we focus on the optimal choice of inflation rate as a target, we omit detailed discussion on gains from policy cooperation in the text. As shown in Appendix A, there are some gains from policy cooperation under both LCP and PCP, as long as $\eta \neq 1$. When $\eta = 1$, there are no gains from policy cooperation because the TOT externality disappears.[13]

5 Conclusion

We analyze optimal monetary policy under the LCP model by comparison with the PCP model. We have two main findings, as follows. We insist that optimal monetary policy under LCP does not stabilize the PPI inflation rate, but does stabilize the CPI inflation rate. In general, optimal monetary policy under LCP involves CPI inflation targeting. This result is quite different from that of Gali and Monacelli (2005). We show that there are no fluctuations in the nominal exchange rate under LCP. Roughly speaking, optimal monetary policy under LCP is consistent with a fixed exchange rate regime, as shown by Devereux and Engel (2003).

[13]Devereux and Engel (2003), assuming Armington consumption, which corresponds to $\eta = 1$ in our model, imply that there are no gains from policy cooperation under both LCP and PCP. As mentioned, there are no gains from policy cooperation when $\eta = 1$ in our model. Hence, our result on policy cooperation is consistent with Devereux and Engel (2003).

Our finding sheds light on Mussa's puzzle, which focuses on co-movements of nominal and real exchange rates, along with Betts and Devereux (2000), although they do not analyze optimal monetary policy. Because complete stabilization in the CPI inflation rate coincides with complete stabilization in the nominal exchange rate under LCP, one of the answers to Mussa's puzzle may be optimal monetary policy under LCP. Solving Mussa's puzzle together with the results of this paper is a task for future research.

Appendix

A Gains from International Monetary Cooperation

By plugging $\eta = \sigma = 1$ into Eqs.(49) and (50) and splitting those equalities following Benigno and Benigno (2006), we have:

$$L^{NC}_{LCP,t} = \frac{\varepsilon}{2\lambda}\pi_t^2 + (1+\varphi)\,x_t^2 \quad ; \quad L^{NC*}_{LCP,t} = \frac{\varepsilon}{2\lambda}\left(\pi_t^*\right)^2 + (1+\varphi)\left(x_t^*\right)^2$$
$$L^{NC}_{PCP,t} = \frac{\varepsilon}{2\lambda}\pi_{P,t}^2 + (1+\varphi)\,x_t^2 \quad ; \quad L^{NC*}_{PCP,t} = \frac{\varepsilon}{2\lambda}\left(\pi_{P,t}^*\right)^2 + (1+\varphi)\left(x_t^*\right)^2,$$

where $L^{NC}_{LCP,t}$ and $L^{NC}_{LCP,t}$ denote period losses under noncooperative settings in both LCP and PCP, respectively.

Under the noncooperative setting, we assume that each central bank minimizes their loss functions as follows, subject to each model:

$$\mathcal{L}^{NC}_{LCP} \equiv \sum_{t=0}^{\infty}\beta^t \mathrm{E}_t\left(L^{NC}_{LCP,t}\right) \;;\; \mathcal{L}^{NC*}_{LCP} \equiv \sum_{t=0}^{\infty}\beta^t \mathrm{E}_t\left(L^{NC*}_{LCP,t}\right)$$
$$\mathcal{L}^{NC}_{PCP} \equiv \sum_{t=0}^{\infty}\beta^t \mathrm{E}_t\left(L^{NC}_{PCP,t}\right) \;;\; \mathcal{L}^{NC*}_{LCP} \equiv \sum_{t=0}^{\infty}\beta^t \mathrm{E}_t\left(L^{NC*}_{LCP,t}\right),$$

as with a cooperative setting. Note that these losses are not measures of welfare costs, but are a minimization problem for central banks under the noncooperative setting.

We solve both cooperative and noncooperative settings analytically, and we plug those solutions into $\mathcal{L}^{W}_{LCP}$ and $\mathcal{L}^{W}_{PCP}$ in the text, respectively. Under PCP, gains from policy cooperation are given by:

$$\mathcal{L}^{NCW}_{PCP} - \mathcal{L}^{W}_{PCP} = \frac{2\left(1+\varphi\right)\Gamma_0^2\left(\sigma+\varphi\right)^2}{\left(1-\beta\right)\omega_3^2}\left[1-\frac{\lambda\varepsilon\left(1+\eta\varphi\right)^2}{\Gamma_1}\right]\left[\text{var}\left(\xi_t\right)+\text{var}\left(\xi_t^*\right)\right],$$

with $\Gamma_0 \equiv \eta^2\left(1+\eta\varphi\right)^2 - \left(1+\eta\varphi\right)$ and $\Gamma_1 \equiv \left(1+\varphi\right)\eta^2 + \lambda\varepsilon\left(1+\eta\varphi\right)^2 > \lambda\varepsilon\left(1+\eta\varphi\right)^2$, hence $1-\frac{\lambda\varepsilon(1+\eta\varphi)^2}{\Gamma_1} > 0$, where $\mathcal{L}^{NCW}_{PCP}\ equiv\frac{1}{2}\left(\mathcal{L}^{NC}_{PCP}+\mathcal{L}^{NC*}_{PCP}\right)$ denotes welfare costs in the PCP model under a noncooperative setting. Under that setting, period loss in country H is given by $L^{NC}_{PCP,t} = \frac{\varepsilon}{2\lambda}\pi_{P,t} + \frac{1+\varphi}{2}x_t^2$ and its counterpart in country F is given analogously. Note that $\Gamma_0 = 0$ when $\eta = 1$. In that case, we have $\mathcal{L}^{NCW}_{PCP} - \mathcal{L}^{W}_{PCP} = 0$, which implies that there are no gains from policy cooperation.

Under LCP, gains from policy cooperation is given by:

$$\mathcal{L}^{NCW}_{LCP} - \mathcal{L}^{W}_{LCP} = \frac{2\left(1+\varphi\right)\left(\sigma+\varphi\right)^2\Gamma_0^2}{\left(1-\beta\right)\omega_3^2}\left[\text{var}\left(\xi_t\right)+\text{var}\left(\xi_t^*\right)\right],$$

where $\mathcal{L}^{NCW}_{LCP} \equiv \frac{1}{2}\left(\mathcal{L}^{NC}_{LCP}+\mathcal{L}^{NC*}_{LCP}\right)$ denotes welfare costs in the LCP model under the noncooperative setting. Under that setting, the period loss in country H is given by $L^{NC}_{LCP,t} = \frac{\varepsilon}{2\lambda}\pi_t + \frac{1+\varphi}{2}x_t^2$ and its counterpart in country F is given analogously. When $\eta = 1$, we have $\mathcal{L}^{NCW}_{LCP} - \mathcal{L}^{W}_{LCP} = 0$, which implies that there are no gains from policy cooperation. Note that we assume discretionary settings and $\rho = 0$ for simplicity in this section.

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Table 1: Macroeconomic Volatility to One-Percent Increase in Productivity

Variables	Pricing	Special ($\sigma = \eta = 1$)		General ($\sigma = 3$, $\eta = 4.5$)	
		a_t	a_t^*	a_t	a_t^*
x_t^W	LCP	0.0000	0.0000	0.0000	0.0000
	PCP	0.0000	0.0000	0.0000	0.0000
π_t^W	LCP	0.0000	0.0000	0.0000	0.0000
	PCP	0.0000	0.0000	0.0000	0.0000
x_t	LCP	0.0000	0.0000	0.0028	0.0028
	PCP	0.0000	0.0013	3.2172e-004	3.2172e-004
x_t^*	LCP	0.0000	0.0000	0.0028	0.0028
	PCP	0.0000	0.0000	3.2172e-004	3.2172e-004
$\pi_{P,t}$	LCP	0.0051	0.0051	0.0011	0.0011
	PCP	0.0000	0.0000	1.0486e-004	1.0486e-004
$\pi_{P,t}^*$	LCP	0.0051	0.0051	0.0011	0.0011
	PCP	0.0000	0.0000	1.0486e-004	1.0486e-004
π_t	LCP	0.0000	0.0000	0.0000	0.0000
	PCP	0.0051	0.0051	0.0012	0.0012
π_t^*	LCP	0.0000	0.0000	0.0000	0.0000
	PCP	0.0051	0.0051	0.0012	0.0012
$\hat{r}_t$	LCP	0.0011	0.0011	0.0023	0.0023
	PCP	0.0021	1.1460e-006	0.0024	0.0020
$\hat{r}_t^*$	LCP	0.0011	0.0011	0.0023	0.0023
	PCP	1.1460e-006	0.0021	0.0020	0.0024
y_t	LCP	0.0229	0.0000	0.0191	0.0038
	PCP	0.0229	0.0000	0.0217	0.0064
y_t^*	LCP	0.0000	0.0229	0.0038	0.0191
	PCP	0.0000	0.0229	0.0064	0.0217
s_t	LCP	0.0229	0.0229	0.0051	0.0051
	PCP	0.0229	0.0229	0.0063	0.0063
z_t	LCP	0.0000	0.0000	0.0000	0.0000
	PCP	0.0000	0.0000	0.0012	0.0012
e_t	LCP	0.0000	0.0000	0.0000	0.0000
	PCP	0.0229	0.0229	0.0057	0.0057

Figure 1: IRFs to Productivity in Country H in the Special Case ($\sigma = \eta = 1$)

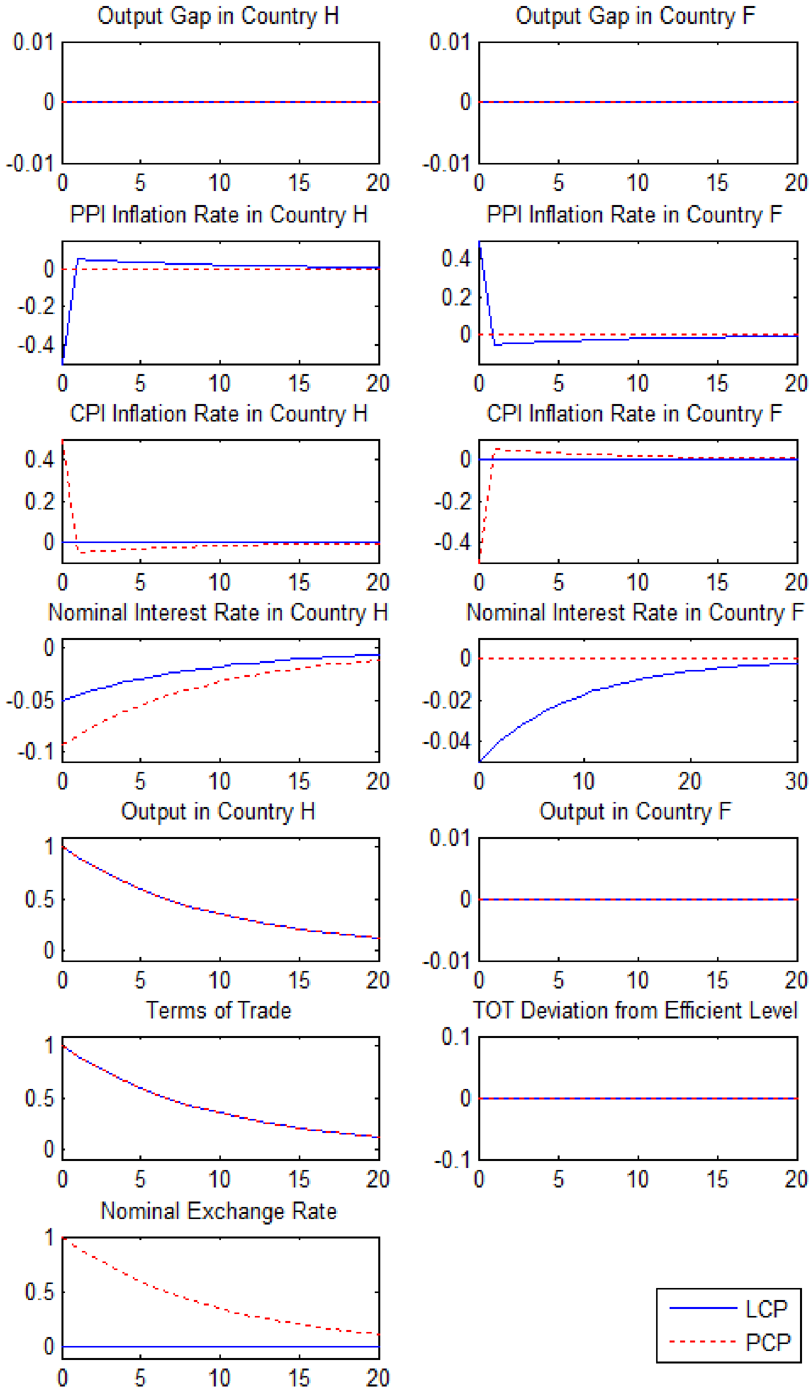

Figure 2: IRFs to Productivity in Country H in the General Case ($\sigma = 3$, $\eta = 4.5$)

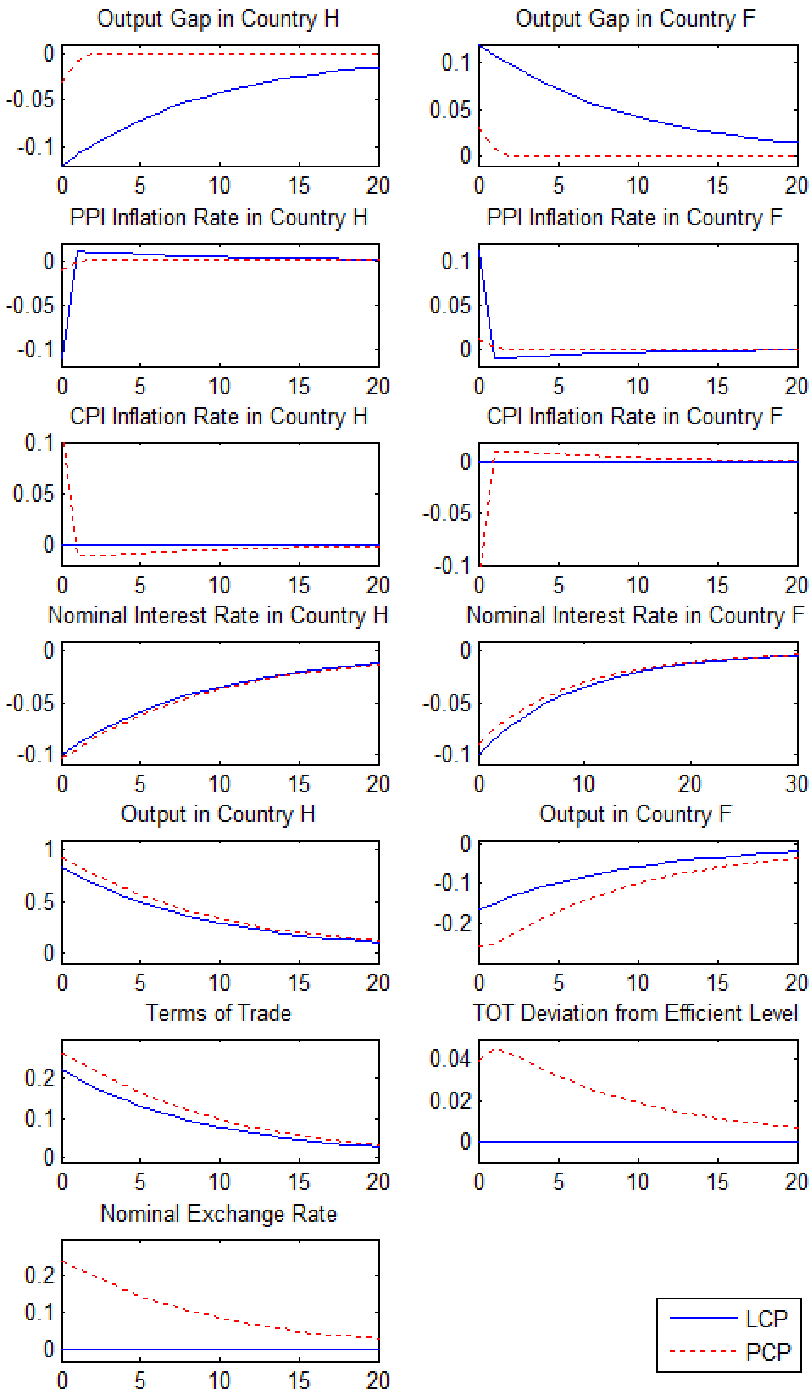

Figure 3: Effects on Macroeconomic Volatilities of Varying Relative Risk Aversion σ and Elasticity of Substitution between Goods Produced in Countries H and F η

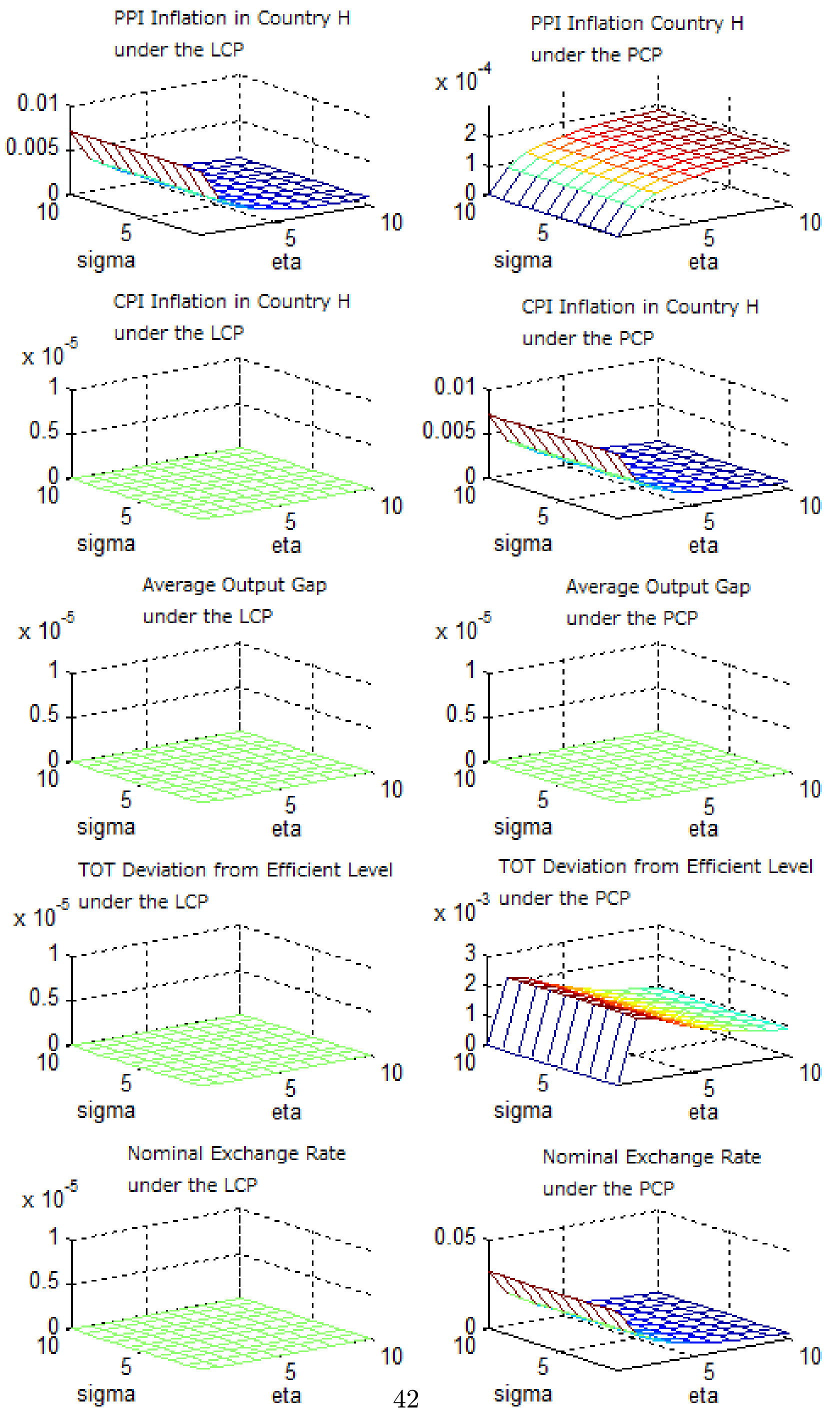

Figure 4: Effects on Welfare Costs of Varying Relative Risk Aversion σ and Elasticity of Substitution between Goods Produced in Countries H and F η