Appendix to: "Optimal Monetary and Fiscal Policy in a Currency Union with Nontradables"

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A Details on Derivation of the Model

In this section, details are provided on the derivation of the model that was developed in section 2 in the text.

A.1 Households

Preferences of the representative household in countries H and F are given by:

$$\begin{aligned}\mathcal{U} &\equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left(\ln C_t - \frac{1}{1+\varphi} N_t^{1+\varphi} \right), \\ \mathcal{U}^* &\equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \delta^t \left[\ln C_t^* - \frac{1}{1+\varphi} \left(N_t^*\right)^{1+\varphi} \right], \qquad \text{(A.1)}\end{aligned}$$

where C_t^* denotes consumption in country F, $N_t^* \equiv N_{F,t} + N_{N,t}^*$ denotes hours of work in country F, $N_{F,t} \equiv \int_1^2 N_{F,t}(f)\, df$ and $N_{N,t}^* \equiv \int_1^2 N_{N,t}^*(f)\, df$ denote hours of work to produce tradables produced in country F and nontradables produced in country F, respectively. The first equality in Eq.(A.1) is Eq.(1) in the text.

More precisely, private consumption is a composite index defined by:

$$\begin{aligned}C_t &\equiv \left[\gamma^{\frac{1}{\eta}} C_{T,t}^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} C_{N,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \\ C_t^* &\equiv \left[\gamma^{\frac{1}{\eta}} \left(C_{T,t}^*\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} \left(C_{N,t}^*\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \qquad \text{(A.2)}\end{aligned}$$

with $C_{N,t} \equiv \left[\int_0^1 C_{N,t}(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}$, $C_{N,t}^* \equiv \left[\int_1^2 C_{N,t}^*(f)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}$ $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}$ and $C_{F,t} \equiv \left[\int_1^2 C_{F,t}(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}$, where the index $\{h, f\}$ denotes a variable that is specific to agents h and f, $C_{T,t}^* = C_{T,t}$ denotes the consumption index for tradables in country F and $C_{N,t}^*$ denotes an index of consumption across the nontradable goods produced in country F. The first equality in Eq.(A.2) is Eq.(2) in the text.

A sequence of budget constraints is given by:

$$\begin{aligned}
B_t + W_t N_t - S_t &\geq \int_0^1 P_{H,t}(h)\, C_{H,t}(h)\, dh + \int_1^2 P_{F,t}(f)\, C_{F,t}(f)\, df \\
&\quad + \int_0^1 P_{N,t}(h)\, C_{N,t}(h)\, dh + \mathrm{E}_t Q_{t,t+1} B_{t+1}, \\
B_t + W_t^* N_t^* - S_t^* &\geq \int_0^1 P_{H,t}(h)\, C_{F,t}^*(f)\, df + \int_1^2 P_{F,t}(f)\, C_{F,t}^*(f)\, df \\
&\quad + \int_0^1 P_{N,t}^*(f)\, C_{N,t}^*(f)\, df + \mathrm{E}_t Q_{t,t+1} B_{t+1}, \qquad \text{(A.3)}
\end{aligned}$$

with $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}$, $P_{F,t} \equiv \left[\int_1^2 P_{F,t}(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$ and $P_{N,t} \equiv \left[\int_0^1 P_{N,t}(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}$, where $P_{N,t}^* \equiv \left[\int_1^2 P_{N,t}^*(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$ denotes the price index of nontradables produced in country F and S_t^* denotes the lump sum taxes in country F. The optimal allocation of any given expenditure within each category of goods yields the following demand functions:

$$\begin{aligned}
C_{H,t}(h) &= \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} C_{H,t}, \quad ; \quad C_{F,t}(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta} C_{F,t}, \\
C_{H,t}^*(h) &= \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} C_{H,t}^*, \quad ; \quad C_{F,t}^*(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta} C_{F,t}^*, \\
C_{N,t}(h) &= \left(\frac{P_{N,t}(h)}{P_{N,t}}\right)^{-\theta} C_{N,t}, \quad ; \quad C_{N,t}^*(f) = \left(\frac{P_{N,t}^*(f)}{P_{N,t}^*}\right)^{-\theta} C_{N,t}^*. \text{(A.4)}
\end{aligned}$$

These equalities implies that $\int_0^1 P_{H,t}(h)\, C_{H,t}(h)\, dh = P_{H,t}C_{H,t}$, $\int_1^2 P_{F,t}(f)\, C_{F,t}(f)\, df = P_{F,t}C_{F,t}$, $\int_0^1 P_{N,t}(h)\, C_{N,t}(h)\, dh = P_{N,t}C_{N,t}$ and $\int_1^2 P_{N,t}^*(f)\, C_{N,t}(f)\, df = P_{N,t}^* C_{N,t}^*$.

Total consumption expenditures by households in countries H and F are given by:

$$\begin{aligned}
P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{N,t}C_{N,t} &= P_t C_t, \\
P_{F,t}C_{F,t} + P_{H,t}C_{H,t} + P_{N,t}^* C_{N,t}^* &= P_t^* C_t^*.
\end{aligned}$$

Combining Eq.(A.3) and these equalities, we obtain:

$$\begin{aligned}
B_t + W_t N_t - S_t &\geq P_t C_t + \mathrm{E}_t Q_{t,t+1} B_{t+1}, \\
B_t + W_t^* N_t^* - S_t^* &\geq P_t^* C_t^* + \mathrm{E}_t Q_{t,t+1} B_{t+1}, \qquad \text{(A.5)}
\end{aligned}$$

where the first equality in Eq.(A.5) is Eq.(3) in the text.

Combining Eq.(A.4) and aggregators, we have:

$$\begin{aligned}
C_{H,t} &= \frac{1}{2}\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-1} C_{T,t}, \quad ; \quad C_{F,t} = \frac{1}{2}\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-1} C_{T,t}, \\
C_{H,t}^* &= \frac{1}{2}\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-1} C_{T,t}^*, \quad ; \quad C_{F,t}^* = \frac{1}{2}\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-1} C_{T,t}^*, \\
C_{T,t} &= \gamma\left(\frac{P_{T,t}}{P_t}\right)^{-\eta} C_t, \quad ; \quad C_{N,t} = (1-\gamma)\left(\frac{P_{N,t}}{P_t}\right)^{-\eta} C_t, \\
C_{T,t}^* &= \gamma\left(\frac{P_{T,t}}{P_t^*}\right)^{-\eta} C_t^*, \quad ; \quad C_{N,t}^* = (1-\gamma)\left(\frac{P_{N,t}^*}{P_t^*}\right)^{-\eta} C_t^*, \qquad \text{(A.6)}
\end{aligned}$$

where the first and the second equalities in Eq.(A.6) make up Eq.(6) in the text.

CPIs are given by:

$$
\begin{aligned}
P_t &\equiv \left[\gamma P_{T,t}^{1-\eta} + (1-\gamma) P_{N,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}, \\
P_t^* &\equiv \left[\gamma P_{T,t}^{1-\eta} + (1-\gamma) \left(P_{N,t}^*\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}, \qquad \text{(A.7)}
\end{aligned}
$$

where P_t^* denotes the CPI in country F. The first equality in Eq.(A.7) is Eq.(4) in the text.

PPIs are given by:

$$
\begin{aligned}
P_{P,t} &\equiv \frac{P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t}}{Y_{H,t} + Y_{N,t}}, \\
P_{P,t}^* &\equiv \frac{P_{F,t}Y_{F,t} + P_{N,t}^*Y_{N,t}^*}{Y_{F,t} + Y_{N,t}^*}, \qquad \text{(A.8)}
\end{aligned}
$$

where $P_{P,t}^*$ denotes the PPI in country F. The first equality in Eq.(A.8) is Eq.(5) in the text.

The representative household maximizes Eq.(A.1) subject to Eq.(A.5). Optimality conditions are given by:

$$
\delta \mathrm{E}_t \frac{C_{t+1}^{-1} P_t}{C_t^{-1} P_{t+1}} = \frac{1}{R_t}, \quad ; \quad \delta \mathrm{E}_t \frac{\left(C_{t+1}^*\right)^{-1} P_t^*}{\left(C_t^*\right)^{-1} P_{t+1}^*} = \frac{1}{R_t}, \qquad \text{(A.9)}
$$

$$
C_t N_t^{\varphi} = \frac{W_t}{P_t}, \quad ; \quad C_t^* \left(N_t^*\right)^{\varphi} = \frac{W_t^*}{P_t^*}. \qquad \text{(A.10)}
$$

The RHS of Eq.(A.9) is an intertemporal optimality condition in country F, whereas the RHS of Eq.(A.10) is an intratemporal optimality condition in country F. The LHS of both Eqs.(A.9) and (A.10) are Eqs.(7) and (8) in the text, respectively.

Combining and iterating Eq.(A.9) with an initial condition, we have the following optimal risk-sharing condition:

$$
C_t = \vartheta C_t^* \mathsf{Q}_t, \qquad \text{(A.11)}
$$

which is Eq.(9) in the text. When $C_{-1} = C_{-1}^* = P_{-1} = P_{-1}^* = 1$, we have $\vartheta = 1$.

A.2 Firms

Each producer can use a linear technology to produce a differentiated good as follows:

$$
\begin{aligned}
Y_{H,t}(h) = A_{H,t} N_{H,t}(h), \quad &; \quad Y_{N,t}(h) = A_{N,t} N_{N,t}(h), \\
Y_{F,t}(f) = A_{F,t} N_{F,t}(f), \quad &; \quad Y_{N,t}^*(f) = A_{N,t}^* N_{N,t}^*(f), \qquad \text{(A.12)}
\end{aligned}
$$

with $Y_{H,t} \equiv \left(\int_0^1 Y_{H,t}(h)^{\frac{\theta-1}{\theta}} dh\right)^{\frac{\theta}{\theta-1}}$, $Y_{F,t} \equiv \left(\int_1^2 Y_{F,t}(f)^{\frac{\theta-1}{\theta}} df\right)^{\frac{\theta}{\theta-1}}$, $Y_{N,t} \equiv \left(\int_0^1 Y_{N,t}(h)^{\frac{\theta-1}{\theta}} dh\right)^{\frac{\theta}{\theta-1}}$ and $Y_{N,t}^* \equiv \left(\int_1^2 Y_{N,t}^*(f)^{\frac{\theta-1}{\theta}} df\right)^{\frac{\theta}{\theta-1}}$, where $A_{F,t}$ and

$A^*_{N,t}$ denote stochastic productivity shifters associated with tradables and nontradables produced in country F, respectively. The first equalities in Eq.(A.12) are Eq.(10) in the text.

Using the Dixit–Stiglitz aggregators, Eq.(A.12) can be rewritten as:

$$
\begin{aligned}
Y_{H,t} &= \frac{A_{H,t} N_{H,t}}{\int_0^1 \frac{Y_{H,t}(h)}{Y_{H,t}} dh} \quad ; \quad Y_{N,t} = \frac{A_{N,t} N_{N,t}}{\int_0^1 \frac{Y_{N,t}(h)}{Y_{N,t}} dh}, \\
Y_{F,t} &= \frac{A_{F,t} N_{F,t}}{\int_0^1 \frac{Y_{F,t}(f)}{Y_{F,t}} df} \quad ; \quad Y^*_{N,t} = \frac{A_{N,t} N^*_{N,t}}{\int_0^1 \frac{Y^*_{N,t}(f)}{Y^*_{N,t}} df}.
\end{aligned}
\tag{A.13}
$$

Under the Calvo–Yun-style price-setting behavior, the pricing rules are given by:

$$
\begin{aligned}
P_{H,t} &= \left[\alpha \left(P_{H,t-1}\right)^{1-\theta} + (1-\alpha)\left(\tilde{P}_{H,t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \\
P_{N,t} &= \left]\alpha \left(P_{N,t-1}\right)^{1-\theta} + (1-\alpha)\left(\tilde{P}_{N,t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \\
P_{F,t} &= \left[\alpha \left(P_{F,t-1}\right)^{1-\theta} + (1-\alpha)\left(\tilde{P}_{F,t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \\
P^*_{N,t} &= \left[\alpha \left(P^*_{N,t-1}\right)^{1-\theta} + (1-\alpha)\left(\tilde{P}^*_{N,t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}},
\end{aligned}
\tag{A.14}
$$

where $\tilde{P}_{F,t}$ and $\tilde{P}^*_{N,t}$ are the prices chosen by firms when they obtain the chance to change prices associated with tradables and nontradables produced in country F, respectively.

The maximization problems faced by firms are as follows:

$$
\begin{aligned}
&\max_{\tilde{P}_{H,t}} \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[\Lambda_{t+k} \tilde{C}_{H,t+k} \left(\tilde{P}_{H,t} - P_{P,t+k} MC_{H,t+k}\right)\right], \\
&\max_{\tilde{P}_{N,t}} \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[\Lambda_{t+k} \tilde{C}_{N,t+k} \left(\tilde{P}_{N,t} - P_{P,t+k} MC_{N,t+k}\right)\right], \\
&\max_{\tilde{P}_{F,t}} \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[\Lambda^*_{t+k} \tilde{C}_{F,t+k} \left(\tilde{P}_{F,t} - P^*_{P,t+k} MC_{F,t+k}\right)\right], \\
&\max_{\tilde{P}^*_{N,t}} \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\delta)^k \left[\Lambda^*_{t+k} \tilde{C}^*_{N,t+k} \left(\tilde{P}^*_{N,t} - P^*_{P,t+k} MC^*_{N,t+k}\right)\right],
\end{aligned}
$$

with $\Lambda_t \equiv (P_t C_t)^{-1}$, $\Lambda^*_t \equiv (P^*_t C^*_t)^{-1}$, $\tilde{C}_{H,t+k} \equiv \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\theta} C_{H,t+k}$ and $\tilde{C}_{N,t+k} \equiv \left(\frac{\tilde{P}_{N,t}}{P_{N,t+k}}\right)^{-\theta} C_{N,t+k}$, where $\tilde{C}_{F,t+k} \equiv \left(\frac{\tilde{P}_{F,t}}{P_{F,t+k}}\right)^{-\theta} C_{F,t+k}$ and $\tilde{C}^*_{N,t+k} \equiv \left(\frac{\tilde{P}^*_{N,t}}{P^*_{N,t+k}}\right)^{-\theta} C^*_{N,t+k}$ denote the total demands when the prices change, $MC_{F,t} \equiv \frac{W^*_t(1-\tau)}{P^*_{P,t} A_{F,t}}$ and $MC^*_{N,t} \equiv \frac{W^*_t(1-\tau)}{P^*_{P,t} A^*_{N,t}}$ denote the marginal costs associated with tradables and nontradables produced in country F, respectively.

The FONCs are as follows:

$$\begin{aligned}
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k \Lambda_{t+k}\tilde{C}_{H,t+k}\left(\tilde{P}_{H,t}-\zeta P_{P,t+k}MC_{H,t+k}\right)\right] &= 0, \\
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k \Lambda_{t+k}\tilde{C}_{N,t+k}\left(\tilde{P}_{N,t}-\zeta P_{P,t+k}MC_{N,t+k}\right)\right] &= 0, \\
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k \Lambda^*_{t+k}\tilde{C}_{F,t+k}\left(\tilde{P}_{F,t}-\zeta P^*_{P,t+k}MC_{F,t+k}\right)\right] &= 0, \\
\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k \Lambda^*_{t+k}\tilde{C}^*_{N,t+k}\left(\tilde{P}^*_{N,t}-\zeta P^*_{P,t+k}MC^*_{N,t+k}\right)\right] &= 0.
\end{aligned} \tag{A.15}$$

The first and second equalities in Eq.(A.15) are Eq.(11) in the text.

Combining the definition of the marginal cost and Eq.(A.10), we have:

$$\begin{aligned}
MC_{H,t} &= \frac{(1-\tau)\,C_t N_t^{\varphi} P_t}{P_{P,t}A_{H,t}}, \quad ; \quad MC_{N,t} = \frac{(1-\tau)\,C_t N_t^{\varphi} P_t}{P_{P,t}A_{N,t}}, \\
MC_{F,t} &= \frac{(1-\tau)\,C_t^*\,(N_t^*)^{\varphi}\,P_t^*}{P^*_{P,t}A_{F,t}}, \quad ; \quad MC^*_{N,t} = \frac{(1-\tau)\,C_t^*\,(N_t^*)^{\varphi}\,P_t^*}{P^*_{P,t}A^*_{N,t}}.
\end{aligned} \tag{A.16}$$

The first equality in Eq.(A.16) is Eq.(12) in the text.

A.3 Centralized Government

The government expenditure index is given by:

$$\begin{aligned}
G_{H,t} &\equiv \left(\int_0^1 G_{H,t}(h)^{\frac{\theta-1}{\theta}}\,dh\right)^{\frac{\theta}{\theta-1}}, \quad ; \quad G_{N,t} \equiv \left(\int_0^1 G_{N,t}(h)^{\frac{\theta-1}{\theta}}\,dh\right)^{\frac{\theta}{\theta-1}}, \\
G_{F,t} &\equiv \left(\int_1^2 G_{F,t}(f)^{\frac{\theta-1}{\theta}}\,df\right)^{\frac{\theta}{\theta-1}} \quad ; \quad G^*_{N,t} \equiv \left(\int_1^2 G^*_{N,t}(f)^{\frac{\theta-1}{\theta}}\,df\right)^{\frac{\theta}{\theta-1}},
\end{aligned}$$

where $G_{F,t}$ denotes government expenditures on tradable goods produced in country F and $G^*_{N,t}$ denotes government expenditures on non-tradable goods produced in country F. For simplicity, we assume that government purchases are fully allocated to a domestically produced good. For any given level of public consumption, the government allocates expenditures across goods in order to minimize total cost. This yields the following set of government demand schedules, analogous to those associated with private consumption.

$$\begin{aligned}
G_{H,t}(h) &= \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} G_{H,t} \quad ; \quad G_{N,t} = \left(\frac{P_{N,t}(h)}{P_{N,t}}\right)^{-\theta} G_{N,t}, \\
G_{F,t}(f) &= \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta} G_{F,t} \quad ; \quad G^*_{N,t} = \left(\frac{P^*_{N,t}(f)}{P^*_{N,t}}\right)^{-\theta} G^*_{N,t}.
\end{aligned} \tag{A.17}$$

Under the optimal monetary policy without a fiscal policy regime, the government expenditure constraint is given by:

$$G_t = G_t^* = 0, \tag{A.18}$$

whereas under the optimal monetary and fiscal policy mix regime, it is relaxed as follows:

$$G_t = -G_t^*. \tag{A.19}$$

A.4 Market Clearing

Market clearing conditions for tradables are given by:

$$\begin{aligned} Y_{H,t}(h) &= C_{H,t}(h) + C_{H,t}^*(h) + G_{H,t}(h), \\ Y_{F,t}(f) &= C_{F,t}(f) + C_{F,t}^*(f) + G_{F,t}(f). \end{aligned} \tag{A.20}$$

The first equality in Eq.(A.20) is Eq.(13) in the text.

As for nontradables, equilibrium requires that:

$$\begin{aligned} Y_{N,t}(h) &= C_{N,t}(h) + G_{N,t}(h), \\ Y_{N,t}^*(f) &= C_{N,t}^*(f) + G_{N,t}^*(f). \end{aligned} \tag{A.21}$$

The first equality in Eq.(A.21) is Eq.(14) in the text.

The market in country H for tradables clears when domestic demand is given by Eq.(A.20). As for nontradables, equilibrium requires Eq.(A.21).

Using Eqs.(A.4), (A.11) and (A.17), Eq.(A.20) can be rewritten as:

$$\begin{aligned} Y_{H,t}(h) &= \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} \left\{\frac{\gamma}{2}\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-1} C_t \left[\left(\frac{P_{T,t}}{P_t}\right)^{-\eta} + \left(\frac{P_{T,t}}{P_t^*}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_{H,t}\right\}, \\ Y_{F,t}(f) &= \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta} \left\{\frac{\gamma}{2}\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-1} C_t \left[\left(\frac{P_{T,t}}{P_t}\right)^{-\eta} + \left(\frac{P_{T,t}}{P_t^*}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_{F,t}\right\}, \end{aligned}$$

where we use the fact that $C_t^* = \frac{C_t}{\mathsf{Q}_t}$, which is derived from Eq.(A.11). Combining these equalities and Eqs.(A.4), (A.11) and (A.17), Eq.(A.20) can be rewritten as:

$$\begin{aligned} Y_{H,t} &= \frac{\gamma}{2}\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-1} C_t \left[\left(\frac{P_{T,t}}{P_t}\right)^{-\eta} + \left(\frac{P_{T,t}}{P_t^*}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_{H,t}, \\ Y_{F,t} &= \frac{\gamma}{2}\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-1} C_t \left[\left(\frac{P_{T,t}}{P_t}\right)^{-\eta} + \left(\frac{P_{T,t}}{P_t^*}\right)^{-\eta} \mathsf{Q}_t^{-1}\right] + G_{F,t}. \end{aligned} \tag{A.22}$$

The first equality in Eq.(A.22) is Eq.(15) in the text.

Using Eqs.(A.4), (A.11) and (A.17), Eq.(A.21) can be rewritten as follows:

$$\begin{aligned} Y_{N,t}(h) &= \left(\frac{P_{N,t}(h)}{P_{N,t}}\right)^{-\theta} \left[(1-\gamma)\left(\frac{P_{N,t}}{P_t}\right)^{-\eta} C_t + G_{N,t}\right], \\ Y_{N,t}^*(f) &= \left(\frac{P_{N,t}(h)}{P_{N,t}}\right)^{-\theta} \left[(1-\gamma)\left(\frac{P_{N,t}^*}{P_t^*}\right)^{-\eta} \frac{C_t}{\mathsf{Q}_t} + G_{N,t}^*\right]. \end{aligned}$$

Combining these equalities and Eqs.(A.4), (A.11) and (A.17), Eq.(A.21) can be rewritten as follows:

$$\begin{aligned} Y_{N,t} &= (1-\gamma)\left(\frac{P_{N,t}}{P_t}\right)^{-\eta} C_t + G_{N,t}, \\ Y^*_{N,t} &= (1-\gamma)\left(\frac{P^*_{N,t}}{P^*_t}\right)^{-\eta} C_t \mathsf{Q}_t^{-1} + G^*_{N,t}. \end{aligned} \tag{A.23}$$

The first equality in Eq.(A.23) is Eq.(16) in the text.

The aggregate domestic indices are defined by:

$$\begin{aligned} Y_t &\equiv \frac{P_{H,t}}{P_{P,t}} Y_{H,t} + \frac{P_{N,t}}{P_{P,t}} Y_{N,t} \quad ; \quad G_t \equiv \frac{P_{H,t}}{P_{G,t}} G_{H,t} + \frac{P_{N,t}}{P_{G,t}} G_{N,t} \\ Y^*_t &\equiv \frac{P_{F,t}}{P^*_{P,t}} Y_{F,t} + \frac{P^*_{N,t}}{P^*_{P,t}} Y^*_{N,t} \quad ; \quad G^*_t \equiv \frac{P_{F,t}}{P^*_{G,t}} G_{F,t} + \frac{P^*_{N,t}}{P^*_{G,t}} G^*_{N,t} \end{aligned} \tag{A.24}$$

The first equality in Eq.(A.24) is Eq.(17) in the text.

B Nonstochastic Steady State

We focus on the equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which the PPI inflation rate is zero. Because this steady state is nonstochastic, all shifters are unit value, i.e., $A_{H,t} = A_{N,t} = A_{F,t} = A^*_{N,t} = D_t = Z_t = 1$. In addition, we assume that $G_t = G^*_t = 0$ in this steady state.

In this steady state, the gross nominal interest rate is equal to the inverse of the subjective discount factor as follows:

$$R = \delta^{-1}.$$

When $\alpha \to 0$, Eq.(A.15) implies that:

$$\begin{aligned} \frac{P_H}{P_P} &= \zeta MC \quad ; \quad \frac{P_N}{P_P} = \zeta MC, \\ \frac{P_F}{P^*_P} &= \zeta MC^* \quad ; \quad \frac{P^*_N}{P^*_P} = \zeta MC^*, \end{aligned}$$

where we use the fact that $MC_H = MC_N \equiv MC$ and $MC_F = MC^*_N \equiv MC^*$ must hold in the nonstochastic steady state. These equalities imply that:

$$\begin{aligned} P_H &= P_N = P_P, \\ P_F &= P^*_N = P^*_P. \end{aligned} \tag{B.1}$$

Eq.(B.1) and FONCs for firms under the nonstochastic steady state imply:

$$MC = MC^* = \zeta^{-1}.$$

In addition,

$$\begin{aligned} MC &= (1-\tau)\, C N^{\varphi} \\ MC^* &= (1-\tau)\, C^* (N^*)^{\varphi}, \end{aligned}$$

is applied. Combining these two conditions associated to marginal cost yields:

$$\frac{\zeta^{-1}}{1-\tau} = CN^{\varphi} = C^{*}\left(N^{*}\right)^{\varphi}$$

To eliminate distortions stemming from a monopolistically competitive market, $\tau = 1-\zeta^{-1}$ is set.

Eq.(A.22) can be rewritten as:

$$\begin{aligned} Y_H &= \frac{\gamma}{2}\left(\frac{P_P^{*}}{P_P}\right)^{\frac{1}{2}} P_T^{-\eta} C\left[P^{\eta} + \left(P^{*}\right)^{\eta} \mathsf{Q}^{-1}\right], \\ Y_F &= \frac{\gamma}{2}\left(\frac{P_P^{*}}{P_P}\right)^{-\frac{1}{2}} P_T^{-\eta} C\left[P^{\eta} + \left(P^{*}\right)^{\eta} \mathsf{Q}^{-1}\right], \end{aligned} \tag{B.2}$$

by using Eq.(B.1). Similar to Eq.(B.2), Eq.(A.23) can be rewritten as:

$$\begin{aligned} Y_N &= (1-\gamma)\left(\frac{P_P}{P}\right)^{-\eta} C, \\ Y_N^{*} &= (1-\gamma)\left(\frac{P_P^{*}}{P^{*}}\right)^{-\eta} C\mathsf{Q}_t^{-1}. \end{aligned} \tag{B.3}$$

Following Gali and Monacelli[1], we assume that the PPP (Purchasing Power Parity) holds in the steady state, which means that:

$$\begin{aligned} \mathsf{Q} &= 1, \\ P_N &= P_N^{*}. \end{aligned} \tag{B.4}$$

Combining Eqs.(B.1) and (B.4), we have:

$$P_P = P_P^{*}. \tag{B.5}$$

Eqs.(B.4), (B.2) and (B.3) imply as follows:

$$\begin{aligned} Y_H = Y_F &= \gamma C, \\ Y_N = Y_N^{*} &= (1-\gamma)\, C. \end{aligned} \tag{B.6}$$

Market clearing in the text implies:

$$\begin{aligned} Y &= Y_H + Y_N \\ Y^{*} &= Y_F + Y_N^{*} \end{aligned}$$

These equalities and Eq.(B.6) imply that:

$$Y = Y^{*} = C. \tag{B.7}$$

Eq.(B.4) imply that:

$$C = C^{*}. \tag{B.8}$$

C Details on Log-linearization of the Model

In this section, which complements section 3 in the text, we describe the details on log-linearization of the model.

C.1 Aggregate Demand and Output

Log-linearizing Eqs.(A.9) and (A.11), we obtain the following:

$$\begin{aligned} c_t &= \mathrm{E}_t c_{t+1} - \hat{r}_t + \mathrm{E}_t \pi_{t+1} + d_t, \\ c_t^R &= \mathsf{q}_t. \end{aligned} \tag{C.1}$$

The second equality in Eq.(C.1) is Eq.(19) in the text.

Log-linearizing and manipulating Eq.(A.7), we obtain:

$$\begin{aligned} \pi_t &= \gamma \pi_{T,t} + (1-\gamma)\, \pi_{N,t}, \\ \pi_t^* &= \gamma \pi_{T,t} + (1-\gamma)\, \pi_{N,t}^*, \end{aligned} \tag{C.2}$$

where $\pi_{N,t}^*$ denotes the inflation rate of nontradables produced in country F. Log-linearizing and manipulating Eq.(A.8), we obtain:

$$\begin{aligned} \pi_{P,t} &= \gamma \pi_{H,t} + (1-\gamma)\, \pi_{N,t}, \\ \pi_{P,t}^* &= \gamma \pi_{F,t} + (1-\gamma)\, \pi_{N,t}^*, \end{aligned} \tag{C.3}$$

where $\pi_{P,t}^*$ denotes the PPI inflation rate in country F.

Log-linearizing Eq.(A.24), we have:

$$\begin{aligned} y_t &= \gamma y_{H,t} + (1-\gamma)\, y_{N,t}, \\ y_t^* &= \gamma y_{F,t} + (1-\gamma)\, y_{N,t}^*. \end{aligned} \tag{C.4}$$

Log-linearizing Eqs.(A.22) and (A.23) and plugging these equalities into Eq.(C.4), we have:

$$\begin{aligned} y_t &= c_t + \frac{\gamma}{2}\mathsf{t}_t + \frac{\psi}{2}\mathsf{n}_t + \hat{g}_t, \\ y_t^* &= c_t^* - \frac{\gamma}{2}\mathsf{t}_t - \frac{\psi}{2}\mathsf{n}_t + \hat{g}_t^*, \end{aligned} \tag{C.5}$$

with $\hat{g}_t^* \equiv \frac{dG_t^*}{Y^*}$ denoting a percentage deviation of government spending from steady-state output levels in country F.

Subtracting the second equality in Eq.(C.5) from the first equality in Eq.(C.5), we have:

$$y_t^R = \gamma \mathsf{t}_t + (1-\gamma)\, \varpi \mathsf{n}_t + \hat{g}_t^R. \tag{C.6}$$

Using the definition of the TOT and Eqs.(C.2) and (C.3), we have:

$$\Delta \mathsf{t}_t = -\frac{1}{\gamma}\pi_{P,t}^R - \frac{1-\gamma}{\gamma}\Delta \mathsf{n}_t. \tag{C.7}$$

Plugging Eqs.(C.7) and (C.5) into Eq.(C.1), we have NKISs as follows:

$$\begin{aligned} y_t &= \mathrm{E}_t y_{t+1} - \hat{r}_t + \mathrm{E}_t \pi_{P,t+1} - \frac{\psi}{2}\Delta \mathrm{E}_t \mathsf{n}_{t+1} - \Delta \mathrm{E}_t \hat{g}_{t+1}, \\ y_t^* &= \mathrm{E}_t y_{t+1}^* - \hat{r}_t + \mathrm{E}_t \pi_{P,t+1}^* + \frac{\psi}{2}\Delta \mathrm{E}_t \mathsf{n}_{t+1} - \Delta \mathrm{E}_t \hat{g}_{t+1}^*. \end{aligned} \tag{C.8}$$

The first equality in Eq.(C.8) is Eq.(20) in the text.

Log-linearizing Eq.(A.19), we have:

$$\hat{g}_t^W = 0,$$

corresponding to Eq.(21) in the text.

Combining this equality and Eq.(22) in the text, we have:

$$\hat{g}_t^W = \hat{g}_t^R$$

which implies:

$$\hat{g}_t = \hat{g}_t^* = 0.$$

Thus, imposing Eqs.(21) and (22) in the text suffices for the necessary and sufficient conditions that government expenditure is zero in each country.

C.2 Aggregate Supply and Inflation

Log-linearizing Eq.(A.15) and rearranging, we can describe the dynamics of inflation in terms of marginal cost as follows:

$$\begin{aligned}
\pi_{H,t} &= \delta \mathrm{E}_t \pi_{H,t+1} + \lambda (1-\gamma) p_{N,t} - \lambda (1-\gamma) p_{H,t} + \lambda mc_{H,t}, \\
\pi_{N,t} &= \delta \mathrm{E}_t \pi_{N,t+1} - \lambda \gamma p_{N,t} + \lambda \gamma p_{H,t} + \lambda mc_{N,t}, \\
\pi_{F,t} &= \delta \mathrm{E}_t \pi_{F,t+1} + \lambda (1-\gamma) p_{N,t}^* - \lambda (1-\gamma) p_{F,t} + \lambda mc_{F,t}, \\
\pi_{N,t}^* &= \delta \mathrm{E}_t \pi_{N,t+1}^* - \lambda \gamma p_{N,t}^* + \lambda \gamma p_{F,t} + \lambda mc_{N,t}^*. \qquad \text{(C.9)}
\end{aligned}$$

Plugging Eq.(C.9) into Eq.(C.3) yields:

$$\begin{aligned}
\pi_{P,t} &= \delta \mathrm{E}_t \pi_{P,t+1} + \lambda mc_t, \\
\pi_{P,t}^* &= \delta \mathrm{E}_t \pi_{P,t+1}^* + \lambda mc_t^*, \qquad \text{(C.10)}
\end{aligned}$$

where:

$$\begin{aligned}
mc_t &= \gamma mc_{H,t} + (1-\gamma) mc_{N,t}, \\
mc_t^* &= \gamma mc_{F,t} + (1-\gamma) mc_{N,t}^*, \qquad \text{(C.11)}
\end{aligned}$$

where mc_t^* denotes the logarithmic domestic marginal cost in country F. The first equality in Eq.(C.10) is Eq.(23) in the text. Eq.(C.11) is derived by log-linearizing the definition of country wide real marginal cost in the text.

Combining the second and fourth equalities of Eq.(C.9), we have:

$$\pi_{N,t}^R = \delta \mathrm{E}_t \pi_{N,t+1}^R + \lambda \gamma \mathsf{n}_t - \lambda \gamma \mathsf{t}_t + \lambda mc_{N,t}^R, \qquad \text{(C.12)}$$

which is Eq.(24) in the text.

Log-linearizing Eq.(A.13) and combining it with Eq.(C.4), we have:

$$\begin{aligned}
y_t &= \gamma a_{H,t} + (1-\gamma) a_{N,t} + n_t, \\
y_t^* &= \gamma a_{F,t} + (1-\gamma) a_{N,t}^* + n_t^*, \qquad \text{(C.13)}
\end{aligned}$$

where we use *Lemma 1* in Appendix E.

Combining log-linearized Eq.(A.10), Eqs.(C.5) and (C.13), we have:

$$
\begin{aligned}
mc_{H,t} &= (1+\varphi)\, y_t - \frac{\psi}{2}\mathsf{n}_t - \hat{g}_t - (1+\varphi\gamma)\, a_{H,t} - (1-\gamma)\,\varphi a_{N,t}, \\
mc_{N,t} &= (1+\varphi)\, y_t - \frac{\psi}{2}\mathsf{n}_t - \hat{g}_t - \varphi\gamma a_{H,t} - [1+(1-\gamma)\,\varphi]\, a_{N,t}, \\
mc_{F,t} &= (1+\varphi)\, y_t^* + \frac{\psi}{2}\mathsf{n}_t - \hat{g}_t^* - (1+\varphi\gamma)\, a_{F,t} - (1-\gamma)\,\varphi a_{N,t}^*, \\
mc_{N,t}^* &= (1+\varphi)\, y_t^* + \frac{\psi}{2}\mathsf{n}_t - \hat{g}_t^* - \varphi\gamma a_{F,t} - [1+(1-\gamma)\,\varphi]\, a_{N,t}^*.
\end{aligned}
\tag{C.14}
$$

Using Eq.(C.11), Eq.(C.14) can be rewritten as follows:

$$
\begin{aligned}
mc_t &= (1+\varphi)\, y_t - \frac{\psi}{2}\mathsf{n}_t - \hat{g}_t - (1+\varphi)\,\gamma a_{H,t} - (1+\varphi)\,(1-\gamma)\, a_{N,t}, \\
mc_t^* &= (1+\varphi)\, y_t^* + \frac{\psi}{2}\mathsf{n}_t - \hat{g}_t^* - (1+\varphi)\,\gamma a_{F,t} - (1+\varphi)\,(1-\gamma)\, a_{N,t}^*.
\end{aligned}
\tag{C.15}
$$

The first equality in Eq.(C.15) is Eq.(25) in the text.

Combining the second and last equalities in Eq.(C.14), we obtain:

$$
\begin{aligned}
mc_{N,t}^R = {} & (1+\varphi)\, y_t^R - \psi\mathsf{n}_t - \hat{g}_t^R - \varphi\gamma a_{H,t} + \varphi\gamma a_{F,t} - [1+(1-\gamma)\,\varphi]\, a_{N,t}, \\
& + [1+(1-\gamma)\,\varphi]\, a_{N,t}^*,
\end{aligned}
\tag{C.16}
$$

which is Eq.(26) in the text.

C.3 Dynamics of Relative Price

Log-linearizing Eq.(A.7) and rearranging it yields:

$$\mathsf{q}_t = (1-\gamma)\,\mathsf{n}_t, \tag{C.17}$$

which is Eq.(18) in the text.

Combining Eqs.(C.1), (C.2) and (C.17), and rearranging, we have:

$$\Delta \mathrm{E}_t \mathsf{n}_{t+1} = \frac{1}{1-\gamma}\Delta \mathrm{E}_t c_{t+1}^R.$$

Combining this equality and Eq.(C.4), we obtain:

$$\Delta \mathrm{E}_t \mathsf{n}_{t+1} = \frac{1}{\psi}\mathrm{E}_t \pi_{P,t+1}^R + \frac{1}{\psi}\Delta \mathrm{E}_t y_{t+1}^R - \frac{1}{\psi}\Delta \mathrm{E}_t \hat{g}_{t+1}^R. \tag{C.18}$$

Using the definition of the NPD and the inflation rate of nontradables, expected changes in the NPD can be written as:

$$\Delta \mathrm{E}_t \mathsf{n}_{t+1} = -\mathrm{E}_t \pi_{N,t+1}^R. \tag{C.19}$$

C.4 Marginal Cost and Output Gap

We define the relationship between output, its natural level and its gap as follows:

$$\begin{aligned} y_t &\equiv \bar{y}_t + \tilde{y}_t, \\ y_t^* &\equiv \bar{y}_t^* + \tilde{y}_t^*, \end{aligned}$$

where $\tilde{y}_t$ denotes the logarithmic output gap at its natural level, and $\bar{y}_t$ denotes the logarithmic natural level output. Under flexible prices, $\tilde{y}_t = \tilde{y}_t^* = 0$ must hold.

When the fiscal authorities design their policies to dissolve the distortion generated by monopolistically competitive markets, real marginal costs under flexible price equilibrium are unity, and their logarithm is given by:

$$mc_t = mc_t^* = 0.$$

In addition, under the flexible price equilibrium, all relative prices are unity. Thus, the logarithmic NPD under the flexible price equilibrium is given by:

$$\mathsf{n}_t = 0.$$

Combining these facts, Eq.(C.15) implies that:

$$\begin{aligned} \bar{y}_t &= \frac{1}{1+\varphi}\hat{g}_t + \gamma a_{H,t} + (1-\gamma)\, a_{N,t}, \\ \bar{y}_t^* &= \frac{1}{1+\varphi}\hat{g}_t^* + \gamma a_{F,t} + (1-\gamma)\, a_{N,t}^*. \end{aligned} \tag{C.20}$$

The first equality in Eq.(C.20) is Eq.(27) in the text.

Using Eq.(C.20), the log-linear approximated model can be rewritten in terms of the output gap. Eq.(C.8) can be rewritten as:

$$\begin{aligned} \tilde{y}_t &= \mathrm{E}_t\tilde{y}_{t+1} - \hat{r}_t + \mathrm{E}_t\pi_{P,t+1} - \frac{\psi}{2}\Delta\mathrm{E}_t\mathsf{n}_{t+1} - \nu\Delta\mathrm{E}_t\hat{g}_{t+1} - \gamma a_{H,t} - (1-\gamma)\, a_{N,t}, \\ \tilde{y}_t^* &= \mathrm{E}_t\tilde{y}_{t+1}^* - \hat{r}_t + \mathrm{E}_t\pi_{P,t+1}^* + \frac{\psi}{2}\Delta\mathrm{E}_t\mathsf{n}_{t+1} - \nu\Delta\mathrm{E}_t\hat{g}_{t+1}^* - \gamma a_{F,t} - (1-\gamma)\, a_{N,t}^*. \end{aligned} \tag{C.21}$$

NKPCs in terms of the output gap are given by:

$$\begin{aligned} \pi_{P,t} &= \delta\mathrm{E}_t\pi_{P,t+1} + \kappa\tilde{y}_t - \frac{\psi\lambda}{2}\mathsf{n}_t, \\ \pi_{P,t}^* &= \delta\mathrm{E}_t\pi_{P,t+1}^* + \kappa\tilde{y}_t^* + \frac{\psi\lambda}{2}\mathsf{n}_t. \end{aligned} \tag{C.22}$$

The first equality in Eq.(C.22) is Eq.(29) in the text.

Similar to NKPCs, we derive NKISs in terms of the output gap. Eq.(C.18) can be rewritten as:

$$\begin{aligned} \Delta\mathrm{E}_t\mathsf{n}_{t+1} &= \frac{1}{\psi}\mathrm{E}_t\pi_{P,t+1}^R + \frac{1}{\psi}\Delta\mathrm{E}_t\tilde{y}_{t+1}^R - \frac{\nu}{\psi}\Delta\mathrm{E}_t\hat{g}_{t+1}^R - \frac{\gamma}{\psi}a_{H,t} + \frac{\gamma}{\psi}a_{F,t}, \\ &\quad - \frac{1-\gamma}{\psi}a_{N,t} + \frac{1-\gamma}{\psi}a_{N,t}^*. \end{aligned} \tag{C.23}$$

Therefore, NKISs are altered as follows:

$$\begin{aligned}\tilde{y}_t &= \mathrm{E}_t\tilde{y}_{t+1} - 2\hat{r}_t + \mathrm{E}_t\pi_{P,t+1} + \mathrm{E}_t\pi^*_{P,t+1} + \Delta\mathrm{E}_t\tilde{y}^*_{t+1} + \bar{r}_t,\\ \tilde{y}^*_t &= \mathrm{E}_t\tilde{y}^*_{t+1} - 2\hat{r}_t + \mathrm{E}_t\pi^*_{P,t+1} + \mathrm{E}_t\pi_{P,t+1} + \Delta\mathrm{E}_t\tilde{y}_{t+1} + \bar{r}_t, \qquad \text{(C.24)}\end{aligned}$$

by plugging Eq.(C.23) into Eq.(C.21). The first equality in Eq.(C.24) is Eq.(28) in the text.

C.5 Canonical Balassa–Samuelson Theorem and NKBS

As mentioned in the former subsection, we now turn to the relationship between the canonical Balassa–Samuelson theorem and the NKBS. Using Eq.(C.20), NKBS Eq.(C.12) can be rewritten as:

$$\pi^R_{N,t} = \delta\mathrm{E}_t\pi^R_{N,t+1} + \lambda\varphi\tilde{y}^R_t + \lambda\mathsf{n}_t + \nu\lambda\hat{g}^R_t - \lambda a_{N,t} + \lambda a^*_{N,t}, \qquad \text{(C.25)}$$

which is Eq.(30) in the text.

D Derivation of the NKPC

The first equality of Eq.(A.15) can be rewritten as:

$$\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\left[\tilde{\mathsf{X}}^{-(\theta-1)}_{H,t+k}\mathsf{X}^{-(\eta-1)}_{T,t+k} - \zeta(1-\tau)\tilde{\mathsf{X}}^{-\theta}_{H,t+k}\mathsf{X}^{-1}_{H,t+k}\mathsf{X}^{-\eta}_{T,t+k}\mathsf{X}_{P,t+k}MC_{H,t+k}\right] = 0,$$

with $\tilde{\mathsf{X}}_{H,t+k} \equiv \frac{\tilde{P}_{H,t}}{P_{H,t+k}}$, $\mathsf{X}_{H,t+k} \equiv \frac{P_{H,t+k}}{P_{T,t+k}}$, $\mathsf{X}_{T,t+k} \equiv \frac{P_{T,t+k}}{P_{t+k}}$ and $\mathsf{X}_{P,t+k} \equiv \frac{P_{P,t+k}}{P_{t+k}}$.

Log-linearizing this equality, we have:

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\tilde{\mathsf{x}}_{H,t+k} + \mathsf{x}_{H,t+k} + \mathsf{x}_{T,t+k} - \mathsf{x}_{P,t+k} - mc_{H,t+k}\right)\right] = 0,$$

with $\tilde{\mathsf{x}}_{H,t+k} \equiv \ln\tilde{\mathsf{X}}_{H,t+k}$, $\mathsf{x}_{H,t+k} \equiv \ln\mathsf{X}_{H,t+k}$, $\mathsf{x}_{T,t+k} \equiv \ln\mathsf{X}_{T,t+k}$ and $\mathsf{x}_{P,t+k} \equiv \ln\mathsf{X}_{P,t+k}$.

Using the fact that $\tilde{\mathsf{x}}_{H,t+k} = \mathsf{x}_{H,t} - \sum_{s=1}^{k}\pi_{H,t+s}$, this can be rewritten as follows:

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\alpha\delta)^k\left(\tilde{\mathsf{x}}_{H,t} - \sum_{s=1}^{k}\pi_{H,t+s} + \mathsf{x}_{H,t+k} + \mathsf{x}_{T,t+k} - \mathsf{x}_{P+k,t} - mc_{H,t+k}\right)\right] = 0.$$

Furthermore, using the fact that $\sum_{k=0}^{\infty}(\alpha\delta)^k\sum_{s=1}^{k}\pi_{H,t+s} = \frac{1}{1-\alpha\delta}\sum_{k=1}^{\infty}(\alpha\delta)^k\pi_{H,t+k}$, this can be rewritten as follows:

$$\begin{aligned}\frac{1}{1-\alpha\delta}\tilde{\mathsf{x}}_{H,t} - \frac{1}{1-\alpha\delta}\mathrm{E}_t\sum_{k=1}^{\infty}(\alpha\delta)^k\pi_{H,t+k} + \mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\mathsf{x}_{H,t+k} + \mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\mathsf{x}_{T,t+k},\\ -\mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k\mathsf{x}_{P+k,t} - \mathrm{E}_t\sum_{k=0}^{\infty}(\alpha\delta)^k mc_{H,t+k} = 0.\end{aligned}$$

Rearranging this, we have:

$$\begin{aligned}\tilde{\mathsf{x}}_{H,t} &= \sum_{k=1}^{\infty}(\alpha\delta)^k \pi_{H,t+k} - (1-\alpha\delta)\sum_{k=0}^{\infty}(\alpha\delta)^k \mathsf{x}_{H,t+k} - (1-\alpha\delta)\sum_{k=0}^{\infty}(\alpha\delta)^k \mathsf{x}_{T,t+k}, \\ &\quad + (1-\alpha\delta)\sum_{k=0}^{\infty}(\alpha\delta)^k \mathsf{x}_{P+k,t} + (1-\alpha\delta)\sum_{k=0}^{\infty}(\alpha\delta)^k mc_{H,t+k}, \\ &= \alpha\delta\pi_{H,t+1} - (1-\alpha\delta)\mathsf{x}_{H,t} - (1-\alpha\delta)\mathsf{x}_{T,t} + (1-\alpha\delta)\mathsf{x}_{P,t} + (1-\alpha\delta)mc_{H,t} + \alpha\delta\tilde{\mathsf{x}}_{H,t+1}.\end{aligned}$$

Log-linearizing Eq.(A.14), we have:

$$\tilde{\mathsf{x}}_{H,t} = \frac{\alpha}{1-\alpha}\pi_{H,t}.$$

Using this fact, we have:

$$\begin{aligned}\pi_{H,t} &= \delta\pi_{H,t+1} - \lambda\mathsf{x}_{H,t} - \lambda\mathsf{x}_{T,t} + \lambda\mathsf{x}_{P,t} + \lambda mc_{H,t}, \\ &= \delta\pi_{H,t+1} + (1-\gamma)\lambda p_{N,t} - (1-\gamma)\lambda p_{H,t} + \lambda mc_{H,t}.\end{aligned}$$

Taking the conditional expectation at t, the second equality in this equation is clearly the same as the first equality in Eq.(C.9). Other NKPCs are derived similarly.

E Welfare Criterion

Following Gali and Monacelli[1] and Woodford[4], we show the derivation of the welfare criterion in the text based on the second-order approximated utility function.

Let $U_t \equiv u(C_t) - v(N_t)$. The second-order Taylor expansion of $u(C_t) \equiv \ln C_t$ is given by:

$$\begin{aligned}u(C_t) &= c_t + \text{t.i.p.} + o\left(\|\xi\|^3\right) \\ &= \tilde{c}_t + \text{t.i.p.} + o\left(\|\xi\|^3\right)\end{aligned} \tag{E.1}$$

with $\tilde{v}_t \equiv v_t - \bar{v}_t$ where $\bar{v}_t$ denotes value of v_t in its flexible price equilibrium.

The second-order Taylor expansion of $v(N_t) \equiv \frac{1}{1+\varphi}N_t^{1+\varphi}$ is as follows:

$$v(N_t) = v(N) + v_N(N)(N_t - N) + \frac{1}{2}v_{NN}(N)(N_t - N)^2 + o\left(\| a \|^3\right), \tag{E.2}$$

with N_t being hours of labor and N being the steady-state value of N_t. Expanding N_t with a second-order Taylor expansion, we have:

$$N_t = N + Nn_t + \frac{1}{2}Nn_t^2 + o\left(\| a \|^3\right). \tag{E.3}$$

Plugging Eq.(E.3) into Eq.(E.2), we obtain:

$$\begin{aligned}v(N_t) &= N^{1+\varphi}\left(n_t + \frac{1+\varphi}{2}n_t^2\right) + \text{t.i.p.} + o\left(\| a \|^3\right) \\ &= N^{1+\varphi}\left(\tilde{n}_t + \frac{1+\varphi}{2}\tilde{n}_t^2\right) + \text{t.i.p.} + o\left(\| a \|^3\right)\end{aligned} \tag{E.4}$$

where we use the fact that $\left(N+Nn_t+\frac{1}{2}Nn_t^2\right)^2 = 2N^2n_t+2N^2n_t^2+\text{t.i.p.}+o\left(\| a \|^3\right)$ and $v_N(N)N = N^{1+\varphi}$.

Combining Eqs.(19), (23), (24) and (35) in the text, we have:

$$c_t = n_t + n_t^* - c_t^* \tag{E.5}$$

Plugging this equality into Eq.(E.1), we have:

$$u(C_t) = \tilde{n}_t + \tilde{n}_t^* - \tilde{c}_t^* \tag{E.6}$$

The optimal allocation must maximize $u(C_t) - v(N_t)$subject to Eq.(E.5). An optimality condition is given by:

$$\frac{\partial\left(u(C_t)-v(N_t)\right)}{\partial N_t} = N_t^{-1} - N_t^{\varphi} = 0.$$

This equality implies that:

$$N_t^{1+\varphi} = 1. \tag{E.7}$$

where we use the technological constraint $y_t = n_t + \text{t.i.p.}$ which is assumed in Gali and Monacelli (2005).

Eq.(A.13) can be rewritten as:

$$N_{H,t} = \frac{Y_{H,t}\int_0^1\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}dh}{A_{H,t}} \quad ; \quad N_{\mathcal{N},t} = \frac{Y_{\mathcal{N},t}\int_0^1\left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta}dh}{A_{\mathcal{N},t}},$$

where we use the facts that $\frac{\int_0^1 Y_{H,t}(h)dh}{Y_{H,t}} = \int_0^1\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}dh$ and $\frac{\int_0^1 Y_{\mathcal{N},t}(h)dh}{Y_{\mathcal{N},t}} = \int_0^1\left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta}dh$.

Log-linearizing these equalities, we obtain:

$$\tilde{n}_{H,t} = \tilde{y}_{H,t} + \ln\int_0^1\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}dh + \text{t.i.p.}, \quad ; \quad \tilde{n}_{\mathcal{N},t} = \tilde{y}_{\mathcal{N},t} + \ln\int_0^1\left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta}dh + \text{t.i.p.}.$$

Combining these equalities, the first equality Eq.(22) in the text and log-linearized definition of country level hours of work, $n_t = \gamma n_{H,t} + (1-\gamma)n_{\mathcal{N},t}$ yields:

$$\tilde{n}_t = \tilde{y}_t + \gamma\ln\int_0^1\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta}dh + (1-\gamma)\ln\int_0^1\left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta}dh + \text{t.i.p.} \tag{E.8}$$

Let define $p_{P,t}(h) \equiv \gamma p_{H,t}(h) + (1-\gamma)p_{\mathcal{N},t}(h)$ and $p_{P,t}^*(f) \equiv \gamma p_{F,t}(f) + (1-\gamma)p_{\mathcal{N},t}^*(f)$ define as the price of domestic goods produced in country H. Eq.(E.8) can be rewritten as:

$$\begin{aligned}
\tilde{n}_t &= \tilde{y}_t + \gamma\ln\mathrm{E}_h\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} + (1-\gamma)\ln\mathrm{E}_h\left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)^{-\theta} + \text{t.i.p.},\\
&= \tilde{y}_t - \theta\mathrm{E}_h\left\{\gamma\ln\left(\frac{P_{H,t}(h)}{P_{H,t}}\right) + \ln(1-\gamma)\left(\frac{P_{\mathcal{N},t}(h)}{P_{\mathcal{N},t}}\right)\right\} + \text{t.i.p.},\\
&= \tilde{y}_t - \theta\mathrm{E}_h\left\{\gamma\left(p_{H,t}(h)-p_{H,t}\right) + (1-\gamma)\left(p_{\mathcal{N},t}(h)-p_{\mathcal{N},t}\right)\right\} + \text{t.i.p.},\\
&= \tilde{y}_t - \theta\ln\mathrm{E}_h\left(\frac{P_{P,t}(h)}{P_{P,t}}\right) + \text{t.i.p.},\\
&= \tilde{y}_t + \ln\int_0^1\left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{-\theta}dh + \text{t.i.p.},
\end{aligned} \tag{E.9}$$

Plugging Eqs.(E.4), (E.6), (E.7) and (E.9) into:

$$\begin{aligned} U_t^W &\equiv \frac{1}{2}\left(U_t + U_t^*\right) \\ &= \frac{1}{2}\left[u\left(C_t\right) - v\left(N_t\right) + u\left(C_t^*\right) - v\left(N_t^*\right)\right], \end{aligned}$$

we have:

$$\begin{aligned} U_t^W &= \frac{1}{2}\left[\tilde{c}_t - N^{1+\varphi}\left(\tilde{n}_t + \frac{1+\varphi}{2}\tilde{n}_t^2\right) + \tilde{c}_t^* - \left(N^*\right)^{1+\varphi}\left(\tilde{n}_t^* + \frac{1+\varphi}{2}\left(\tilde{n}_t^*\right)^2\right)\right] \\ &\quad + \text{t.i.p.} + o\left(\|\xi\|^3\right) \\ &= -\frac{1}{2}\left[\ln\int_0^1\left(\frac{P_{P,t}\left(h\right)}{P_{P,t}}\right)dh + \frac{1+\varphi}{2}\tilde{n}_t^2 + \ln\int_1^2\left(\frac{P_{P,t}^*\left(f\right)}{P_{P,t}^*}\right)df + \frac{1+\varphi}{2}\left(\tilde{n}_t^*\right)^2\right] \\ &\quad + \text{t.i.p.} + o\left(\|\xi\|^3\right). \end{aligned}$$

Note that we use Eq.(E.5) to eliminate linear terms. Combining this equality, Eq.(C.13) and the definition of output gap in the text, we have:

$$\begin{aligned} U_t^W &= -\frac{1}{2}\left[\ln\int_0^1\left(\frac{P_{P,t}\left(h\right)}{P_{P,t}}\right)dh + \frac{1+\varphi}{2}\tilde{y}_t^2 + \ln\int_1^2\left(\frac{P_{P,t}^*\left(f\right)}{P_{P,t}^*}\right)df + \frac{1+\varphi}{2}\left(\tilde{y}_t^*\right)^2\right] \\ &\quad + \text{t.i.p.} + o\left(\|\xi\|^3\right). \end{aligned} \tag{E.10}$$

Let define $\hat{p}_{P,t}(h) \equiv p_{P,t}(h) - p_{P,t}$. As derived by Gali and Monacelli[1], notice that:

$$\begin{aligned} \left(\frac{P_{P,t}\left(h\right)}{P_{P,t}}\right)^{1-\theta} &= \exp\left[\left(1-\theta\right)\hat{p}_{P,t}\left(h\right)\right] \\ &= 1 - \left(1-\theta\right)\hat{p}_{P,t}\left(h\right) + \frac{\left(1-\theta\right)^2}{2}\hat{p}_{P,t}\left(h\right) + o\left(\|\xi\|^3\right) \end{aligned} \tag{E.11}$$

In the symmetric equilibrium, we have $\frac{P_{P,t}(h)}{P_{P,t}} = 1$. This implies:

$$\mathrm{E}_h\left(\frac{P_{P,t}\left(h\right)}{P_{P,t}}\right)^{1-\theta} = 1 \tag{E.12}$$

Combining Eqs.(E.11) and (E.12), we have:

$$\mathrm{E}_h\hat{p}_{P,t}\left(h\right) = \frac{\theta-1}{2}\mathrm{E}_h\hat{p}_{P,t}\left(h\right)^2 \tag{E.13}$$

In addition, second order approximation to $\left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{-\theta}$, yields:

$$\left(\frac{P_{P,t}\left(h\right)}{P_{P,t}}\right)^{-\theta} = 1 - \theta\hat{p}_{P,t}\left(h\right) + \frac{\theta^2}{2}\hat{p}_{P,t}\left(h\right)^2 + o\left(\|\xi\|^3\right)$$

This equality implies as follows:

$$\int_0^1\left(\frac{P_{P,t}\left(h\right)}{P_{P,t}}\right)^{-\theta}dh = 1 - \theta\mathrm{E}_h\hat{p}_{P,t}\left(h\right) + \frac{\theta^2}{2}\mathrm{E}_h\hat{p}_{P,t}\left(h\right)^2 + o\left(\|\xi\|^3\right) \tag{E.14}$$

Plugging Eq.(E.13) into Eq.(E.14), we have:

$$\begin{aligned}\int_0^1 \left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{-\theta} dh &= 1+\frac{\theta}{2}\mathrm{E}_h \hat{p}_{P,t}(h)^2 + o\left(\|\xi\|^3\right) \\ &= 1+\frac{\theta}{2}\mathrm{var}_h\left(\hat{p}_{P,t}(h)\right) + o\left(\|\xi\|^3\right)\end{aligned}$$

This equality imples as follows:

$$\ln\int_0^1 \left(\frac{P_{P,t}(h)}{P_{P,t}}\right)^{-\theta} dh = \frac{\theta}{2}\mathrm{var}_h\left(p_{P,t}(h)\right) + o\left(\|\xi\|^3\right) \qquad \text{(E.15)}$$

which clearly corresponds to the one derived by Gali and Monacelli[1].

Lemma 1

$$\sum_{t=0}^{\infty}\delta^t \mathrm{var}_h\left(p_{P,t}(h)\right) = \frac{1}{\lambda}\sum_{t=0}^{\infty}\delta^t \pi_{P,t}^2$$

Proof: See Woodford[4], p 399-400.

Combining *Lemma 1*, Eqs.(E.10) and (E.15), we have:

$$\begin{aligned}\mathcal{U}^W &= \mathrm{E}_0\sum_{t=0}^{\infty}\delta^t U_t^W \\ &= -\mathrm{E}_0\sum_{t=0}^{\infty}\delta^t\frac{1}{4}\left[\frac{\theta}{\lambda}\pi_{P,t}^2 + (1+\varphi)\tilde{y}_t^2 + \frac{\theta}{\lambda}\left(\pi_{P,t}^*\right)^2 + (1+\varphi)\left(\tilde{y}_t^*\right)^2\right] \\ &\quad + \text{t.i.p.} + o\left(\|\xi\|^3\right) \\ &= -\mathrm{E}_0\sum_{t=0}^{\infty}\delta^t L_t + \text{t.i.p.} + o\left(\|\xi\|^3\right),\end{aligned}$$

where $L_t^W \equiv \frac{1}{4}\left[\frac{\theta}{\lambda}\pi_{P,t}^2 + (1+\varphi)\tilde{y}_t^2 + \frac{\theta}{\lambda}\left(\pi_{P,t}^*\right)^2 + (1+\varphi)\left(\tilde{y}_t^*\right)^2\right]$. This corresponds to Eq.(33) in the text.

F The Optimal Monetary Policy Rule

The central bank seeks to minimize Eq.(E.1) subject to Eqs.(C.22), (C.24) and (C.25). The Lagrangian is given by:

$$\pounds = \mathrm{E}_0\sum_{t=0}^{\infty}\delta^t 2\left[\begin{array}{c} L_t^W + \mu_{1,t}\left(\tilde{y}_t - \tilde{y}_{t+1} + 2\hat{r}_t - \pi_{P,t+1} - \pi_{P,t+1}^* - \Delta\tilde{y}_{t+1}^*\right) \\ +\mu_{2,t}\left(\tilde{y}_t^* - \tilde{y}_{t+1}^* + 2\hat{r}_t - \pi_{P,t+1}^* - \pi_{P,t+1} - \Delta\tilde{y}_{t+1}\right) \\ +\mu_{3,t}\left(\mathsf{n}_{t+1} - \mathsf{n}_t - \pi_{N,t+1}^R\right) \\ +\mu_{4,t}\left(\pi_{P,t} - \delta\pi_{P,t+1} - \kappa\tilde{y}_t + \frac{\psi\lambda}{2}\mathsf{n}_t\right) \\ +\mu_{5,t}\left(\pi_{P,t}^* - \delta\pi_{P,t+1}^* - \kappa\tilde{y}_t^* - \frac{\psi\lambda}{2}\mathsf{n}_t\right) \\ +\mu_{6,t}\left(\pi_{N,t}^R - \delta\pi_{N,t+1}^R + \varphi\lambda\tilde{y}_t - \varphi\lambda\tilde{y}_t^* + \lambda\mathsf{n}_t\right) \end{array}\right].$$

FONCs associated with this Lagrangian with respect to r_t, $\pi_{P,t}$, $\pi_{P,t}^*$, $\tilde{y}_t$, $\tilde{y}_t^*$, n_t and $\pi_{N,t}^R$ are given by:

$$\mu_{1,t} + \mu_{2,t} = 0,$$

$$
\begin{aligned}
\frac{\theta}{\lambda}\pi_{P,t}+\mu_{4,t} &= 0,\\
\frac{\theta}{\lambda}\pi^*_{P,t}+\mu_{5,t} &= 0,\\
(1+\varphi)\,\tilde{y}_t+\mu_{1,t}+\mu_{2,t}-\kappa\mu_{4,t}+\varphi\lambda\mu_{6,t} &= 0,\\
(1+\varphi)\,\tilde{y}^*_t+\mu_{1,t}+\mu_{2,t}-\kappa\mu_{5,t}+\varphi\lambda\mu_{6,t} &= 0,\\
\mu_{6,t} &= 0,\\
-\mu_{3,t}+\frac{\psi\lambda}{2}\mu_{4,t}-\frac{\psi\lambda}{2}\mu_{5,t}+\lambda\mu_{6,t} &= 0.
\end{aligned}
$$

Rearranging the FONCs, we obtain:

$$
\begin{aligned}
\tilde{y}_t &= -\theta\pi_{P,t},\\
\tilde{y}^*_t &= -\theta\pi^*_{P,t}.
\end{aligned}
\tag{F.1}
$$

Combining both the first and second equalities in Eq.(F.1) yields:

$$
\tilde{y}^W_t=-\theta\pi^W_t. \tag{F.2}
$$

Following Monacelli[2], let the NKPCs be as follows:

$$
\begin{aligned}
\pi_{P,t} &= \delta\mathrm{E}_t\pi_{P,t+1}+\kappa\tilde{y}_t-\frac{\psi\lambda}{2}\mathsf{n}_t+\varepsilon_t,\\
\pi^*_{P,t} &= \delta\mathrm{E}_t\pi^*_{P,t+1}+\kappa\tilde{y}^*_t+\frac{\psi\lambda}{2}\mathsf{n}_t+\varepsilon^*_t,
\end{aligned}
\tag{F.3}
$$

where ε_t and ε^*_t denote supply shocks that prevent the central bank from simultaneously stabilizing inflation and the output gap in the equalities of countries H and F, respectively.[1]

Plugging Eq.(F.1) into Eq.(F.3), we have:

$$
\begin{aligned}
\pi_{P,t} &= \frac{\delta}{1+\kappa\theta}\mathrm{E}_t\pi_{P,t+1}-\frac{\psi\lambda}{(1+\kappa\theta)\,2}\mathsf{n}_t+\frac{1}{1+\kappa\theta}\varepsilon_t,\\
\pi^*_{P,t} &= \frac{\delta}{1+\kappa\theta}\mathrm{E}_t\pi^*_{P,t+1}+\frac{\psi\lambda}{(1+\kappa\theta)\,2}\mathsf{n}_t+\frac{1}{1+\kappa\theta}\varepsilon^*_t.
\end{aligned}
$$

Iterating forward, we have:

$$
\begin{aligned}
\pi_{P,t} &= \frac{1}{1+\kappa\theta}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\kappa\theta}\right)^j\varepsilon_{t+j}-\frac{\psi\lambda}{(1+\kappa\theta)\,2}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\kappa\theta}\right)^j\mathsf{n}_{t+j},\\
\pi^*_{P,t} &= \frac{1}{1+\kappa\theta}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\kappa\theta}\right)^j\varepsilon^*_{t+j}+\frac{\psi\lambda}{(1+\kappa\theta)\,2}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\kappa\theta}\right)^j\mathsf{n}_{t+j},
\end{aligned}
$$

where we use the fact that $\frac{\delta}{1+\kappa\theta}<1$. Taking conditional expectations at t, this can be altered as follows:

$$
\begin{aligned}
\pi_{P,t} &= \frac{1}{1+\kappa\theta}\varepsilon_t-\frac{\psi\lambda}{(1+\kappa\theta)\,2}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\kappa\theta}\right)^j\mathsf{n}_{t+j},\\
\pi^*_{P,t} &= \frac{1}{1+\kappa\theta}\varepsilon^*_t+\frac{\psi\lambda}{(1+\kappa\theta)\,2}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\kappa\theta}\right)^j\mathsf{n}_{t+j}.
\end{aligned}
\tag{F.4}
$$

[1] Although ε_t does not appear explicitly in our model, this is introduced to derive an optimal policy rule. See Monacelli[2] for details.

Combining Eqs.(F.2) and (F.4), we have:

$$\pi_t^W = \frac{1}{1+\kappa\theta}\varepsilon_t^W \ ; \ \tilde{y}_t^W = -\frac{\theta}{1+\kappa\theta}\varepsilon_t^W,$$

which implies:

$$\mathrm{E}_t\pi_{t+1}^W = \mathrm{E}_t\tilde{y}_{t+1}^W = 0. \tag{F.5}$$

Combining the first and second equalities in Eq.(C.24), we obtain:

$$\tilde{y}_t^W = \mathrm{E}_t\tilde{y}_{t+1}^W - 2\hat{r}_t + \mathrm{E}_t\pi_{t+1}^W + \bar{r}_t. \tag{F.6}$$

Plugging Eqs.(F.2) and (F.5) into Eq.(F.6) yields:

$$\hat{r}_t = \frac{1}{2}\bar{r}_t + \frac{\theta}{2}\pi_{P,t} + \frac{\theta}{2}\pi_{P,t}^*.$$

This equality is Eq.(35) in the text.

G The Optimal Fiscal Policy Rule

The centralized government seeks to minimize Eq.(E.1) subject to Eqs.(C.22), (C.24) and (C.25). The Lagrangian is given by:

$$\pounds = \mathrm{E}_0\sum_{t=0}^{\infty}\delta^t 2\left[\begin{array}{c} L_t^W + \mu_{1,t}\left(\tilde{y}_t - \tilde{y}_{t+1} + 2\hat{r}_t - \pi_{P,t+1} - \pi_{P,t+1}^* - \Delta\tilde{y}_{t+1}^*\right) \\ +\mu_{2,t}\left(\tilde{y}_t^* - \tilde{y}_{t+1}^* + 2\hat{r}_t - \pi_{P,t+1}^* - \pi_{P,t+1} - \Delta\tilde{y}_{t+1}\right) \\ +\mu_{3,t}\left(\mathsf{n}_{t+1} - \mathsf{n}_t - \pi_{N,t+1}^R\right) \\ +\mu_{4,t}\left(\pi_{P,t} - \delta\pi_{P,t+1} - \kappa\tilde{y}_t + \frac{\psi\lambda}{2}\mathsf{n}_t\right) \\ +\mu_{5,t}\left(\pi_{P,t}^* - \delta\pi_{P,t+1}^* - \kappa\tilde{y}_t^* - \frac{\psi\lambda}{2}\mathsf{n}_t\right) \\ +\mu_{6,t}\left(\pi_{N,t}^R - \delta\pi_{N,t+1}^R + \varphi\lambda\tilde{y}_t - \varphi\lambda\tilde{y}_t^* + \lambda\mathsf{n}_t + \nu\lambda g_t^R\right) \end{array}\right].$$

The FONCs associated with this Lagrangian with respect to $\pi_{P,t}$, $\pi_{P,t}^*$, $\tilde{y}_t$, $\tilde{y}_t^*$, n_t, $\pi_{N,t}^R$ and g_t^R are given by:

$$\begin{aligned} \frac{\theta}{\lambda}\pi_{P,t} + \mu_{4,t} &= 0, \\ \frac{\theta}{\lambda}\pi_{P,t}^* + \mu_{5,t} &= 0, \\ (1+\varphi)\,\tilde{y}_t + \mu_{1,t} + \mu_{2,t} - \kappa\mu_{4,t} + \varphi\lambda\mu_{6,t} &= 0, \\ (1+\varphi)\,\tilde{y}_t^* + \mu_{1,t} + \mu_{2,t} - \kappa\mu_{5,t} + \varphi\lambda\mu_{6,t} &= 0, \\ \mu_{6,t} &= 0, \\ -\mu_{3,t} + \frac{\psi\lambda}{2}\mu_{4,t} - \frac{\psi\lambda}{2}\mu_{5,t} + \lambda\mu_{6,t} &= 0, \\ \mu_{6,t} &= 0. \end{aligned}$$

Rearranging the FONCs, we obtain:

$$\theta\pi_{P,t} + \tilde{y}_t = \theta\pi_{P,t}^* + \tilde{y}_t^*.$$

This implies that:

$$\tilde{y}_t^R = -\theta \pi_{P,t}^R. \tag{G.1}$$

Combining both equalities in Eq.(F.3), we have:

$$\pi_{P,t}^R = \delta \mathrm{E}_t \pi_{P,t+1}^R + \kappa \tilde{y}_t^R - \psi\lambda \mathsf{n}_t + \varepsilon_t^R,$$

with $\varepsilon_t^R \equiv \varepsilon_t - \varepsilon_t^*$. Using Eq.(G.1), Eq.(G.2) can be rewritten as:

$$\pi_{P,t}^R = \frac{\delta}{1+\kappa\theta} \mathrm{E}_t \pi_{P,t+1}^R - \psi\lambda \mathsf{n}_t + \varepsilon_t^R. \tag{G.2}$$

Iterating Eq.(G.2) forward yields:

$$\pi_{P,t}^R = \frac{1}{1+\kappa\theta}\varepsilon_t^R - \frac{\psi\lambda}{1+\kappa\theta}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\kappa\theta}\right)^j \mathsf{n}_{t+j}. \tag{G.3}$$

Putting Eq.(G.3) forward one period and taking the conditional expectation at period t, we have:

$$\mathrm{E}_t \pi_{P,t+1}^R = -\frac{\psi\lambda}{1+\kappa\theta}\sum_{j=0}^{\infty}\left(\frac{\delta}{1+\kappa\theta}\right)^j \mathsf{n}_{t+1+j}. \tag{G.4}$$

Plugging Eqs.(G.3) and (G.4) into Eq.(G.2) yields:

$$\mathsf{n}_t = \frac{1}{\psi\lambda}\varepsilon_t^R. \tag{G.5}$$

This implies:

$$\begin{aligned} \mathrm{E}_t \pi_{N,t+1}^R &= -\mathrm{E}_t \mathsf{n}_{t+1} + \mathsf{n}_t \\ &= \frac{1}{\psi\lambda}\varepsilon_t^R. \end{aligned} \tag{G.6}$$

Plugging Eqs. (G.1), (G.5) and (G.6) into Eq.(C.25), we obtain:

$$\hat{g}_t^R = \bar{g}_t^R + \theta_\sigma \pi_{P,t}^R + \frac{1}{\nu\lambda}\pi_{N,t}^R - \frac{\lambda+\delta}{\nu\lambda}\mathsf{n}_t.$$

This is Eq.(37) in the text.

H Derivation for Explicit Social Loss

The local government in countries H and F seek to minimize their respective losses $\mathcal{L}^{NC}$ and $\mathcal{L}^{NC*}$ subject to structural model. FONCs imply as follows:

$$\begin{aligned} \pi_{P,t}^W &= \frac{1}{1+\kappa\theta}\varepsilon_t^W \quad ; \quad \tilde{y}_t^W = -\frac{\theta}{1+\kappa\theta}\varepsilon_t^W \\ \pi_{P,t}^R &= \left(\frac{1}{1+\kappa\theta}\right)^2 \varepsilon_t^R \quad ; \quad \tilde{y}_t^R = -\theta\left(\frac{1}{1+\kappa\theta}\right)^2 \varepsilon_t^R \end{aligned} \tag{H.7}$$

Note that we combine and iterate FONCs. The centralized government seeks to minimize its union-wide losses $\mathcal{L}^W$. Equalities derived by FONCs corresponds to Eq.(H.7).

Plugging Eq.(H.7) into Eq.(38) in the text, we have:

$$\begin{aligned} L_t^{NC} &= \frac{1}{2}\left\{\left[\frac{1}{(1+\kappa\theta)\,2}\right]^2\left(1+\frac{1}{1+\kappa\theta}\right)^2\left[\frac{\theta}{\kappa}+(1+\varphi)\,\theta^2\right]\varepsilon_t^2\right. \\ &\quad \left.+\left[\frac{1}{(1+\kappa\theta)\,2}\right]^2\left(1+\frac{1}{1+\kappa\theta}\right)^2\left[\frac{\theta}{\kappa}+(1+\varphi)\,\theta^2\right](\varepsilon_t^*)^2\right\} \end{aligned}$$

Because of constant variance, this equality implies as follows:

$$\begin{aligned} \mathcal{L}_t^{NC} &= \frac{1}{2}\left\{\left[\frac{1}{(1+\kappa\theta)\,2}\right]^2\left(1+\frac{1}{1+\kappa\theta}\right)^2\left[\frac{\theta}{\kappa}+(1+\varphi)\,\theta^2\right]\sigma_\varepsilon^2\right. \\ &\quad \left.+\left[\frac{1}{(1+\kappa\theta)\,2}\right]^2\left(1+\frac{1}{1+\kappa\theta}\right)^2\left[\frac{\theta}{\kappa}+(1+\varphi)\,\theta^2\right](\sigma_\varepsilon^*)^2\right\} \end{aligned} \quad \text{(H.8)}$$

Plugging Eq.(H.8) and its counterpart in country F into the definition of the union-wide social loss brought about by self-oriented fiscal authorities $\mathcal{L}^{NCW}$, we have the equality in page 29 in the text. Plugging Eq.(H.7) into the expected welfare loss function $\mathcal{L}^W$, we have the same equality, namely, the equality in page 29 in the text. Thus, $\mathcal{L}^W = \mathcal{L}^{NCW}$ holds.

References

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[4] Woodford, Michael (2001), "Inflation Stabilization and Welfare," *NBER Working Paper*, No. 8071.